# Portfolio Choice, Military Protection, and Exorbitant Privilege*

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**Abstract**

We develop a two-country model in which an Ally is vulnerable to an external aggression, while a military hegemon (the U.S.) has the capacity to prevent or mitigate the consequences of such aggression. We show that in this environment, the Ally may find it optimal to "buy military protection, or a security guarantee" from the United States by subsidizing U.S. foreign direct investments at home (FDIs) and domestic residents' purchases of U.S. bonds. The U.S. security guarantee comes at a price for the Ally: in the allocation under the Ally's optimal capital account policy, the United States earns an excess return on its international investment position, which strengthens the dollar and helps finance a U.S. trade deficit. This highlights why such an equilibrium is also optimal from the perspective of the hegemon and how it contributes to the U.S. exorbitant privilege and global imbalances. The paper also provides empirical evidence on the critical predictions of the model.

**Keywords:** Defense expenditure, Exorbitant privilege, FDIs, Global imbalances, Wars, Portfolio Choice.
**JEL Classification:** D74, F36, G11, H56.

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*We should therefore expect a return to the Cold War logic in which economic relations more frequently align with security relations. With the United States at the bullseye of the largest security network in the world, the dollar stands to benefit from this shift. As during the Cold War, US security provision may induce allied dollar support.*" Carla Norrlöf, written testimony to the U.S. House Financial Services Subcommittee hearing "Dollar Dominance: Preserving the U.S. Dollar's Status as the Global Reserve Currency", June 7, 2023.

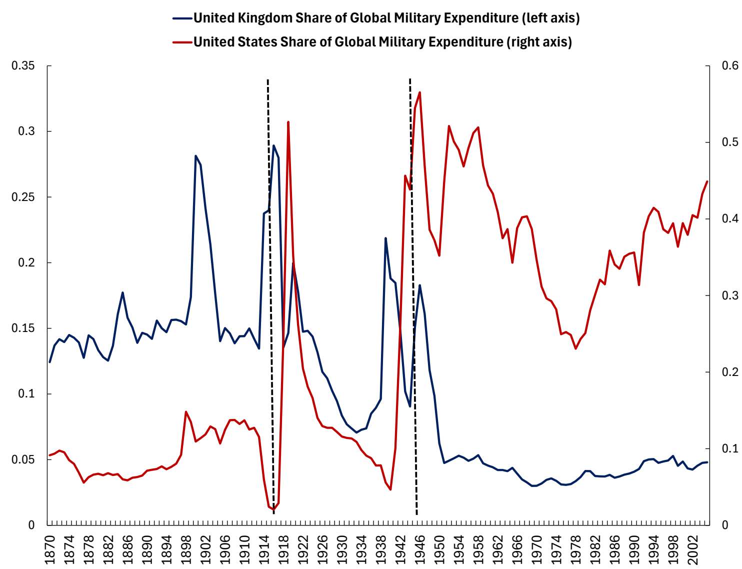
*America provides a global defense shield to liberal democracies, and in exchange, America receives the benefits of reserve status—and, as we are grappling with today, the burdens.*" Stephen Miran, A User's Guide to Restructuring the Global Trading System, November 2024.

# 1 Introduction

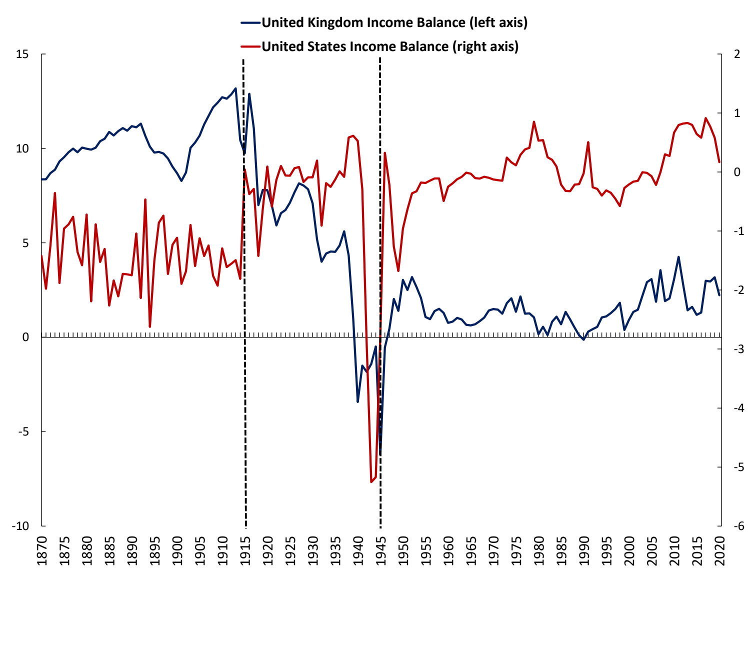
The notion that military power, global security provision, and global reserve-currency status are mutually reinforcing has gained traction in policy and academic debates (e.g., Norrlöf, 2023, Miran, 2024). Three stylized facts are consistent with this perspective: (i) the military hegemon historically earns an excess return on its international investment position—the "exorbitant privilege"; (ii) it serves as the world's largest net exporter of physical capital; and (iii) it bears a disproportionate share of the collective defense burden, while its allies partially free ride on these efforts. In this paper, we develop a general equilibrium model that rationalizes these three facts as the outcome of an optimal arrangement between a military hegemon and an ally. When faced with a security threat, the ally effectively "buys" a security guarantee from the hegemon by subsidizing the hegemon's foreign direct investments (FDIs) within its borders and domestic residents' purchases of the hegemon's bond. We further provide empirical evidence demonstrating the link between FDIs and U.S. security provision, which is at the core of our model.

**Figure 1** Military Hegemony and Exorbitant Privilege: The United Kingdom and The United States (1870-2005)

Panel (A)



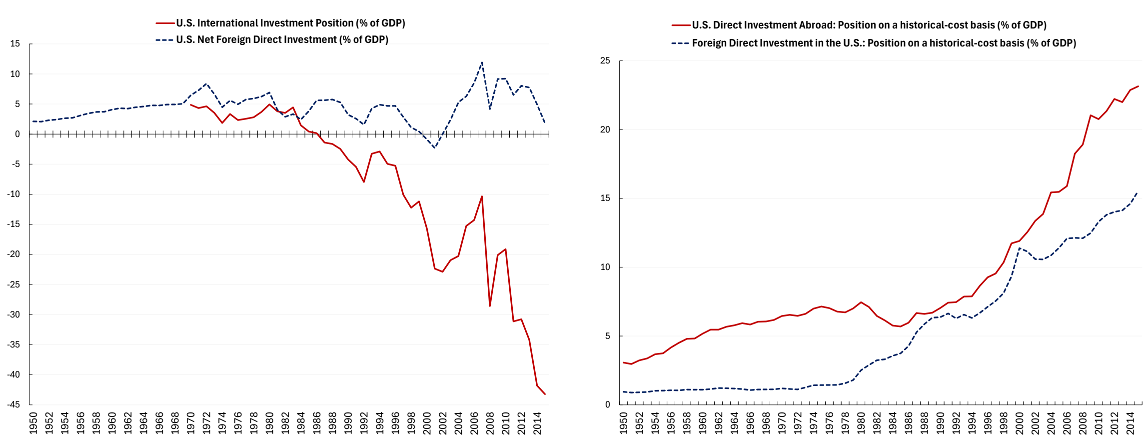

Panel (B)




Note. The figure plots the British (dark, blue solid line) and American (red solid line) shares of world defense expenditure (Panel A) and income balance to GDP from 1970 to 2005 (Panel B). The income balance is the difference between the current account of the balance of payments and the trade balance in goods and services. The main data sources are the Global Military Spending Dataset (GMSD) of Barnum, Fariss, Markowitz and Morales (2024) and the Macrohistory Database of Jordà, Schularick and Taylor (2017). See Data Appendix for all variable definitions and more details on the sources. With respect to typical measures of exorbitant privilege, the income balance omits valuation effects, which are available only from 1970 onward.

Over the past century, military hegemony has shifted from the United Kingdom (U.K.) to the United States (U.S.). Figure 1 illustrates this transition, plotting the British and American shares of world defense expenditure from 1870 to 2005 (Panel A). During the interwar period (1915–1945), marked by the two vertical lines, the United States overtook the United Kingdom to emerge as the world's military hegemon. In 1915, the U.K.'s share of global military expenditures was approximately 1.5 times larger than that of the U.S.; by 2005, however, the U.S. share had grown to nearly ten times that of the U.K. At the same time, British income from foreign investments—including interest and dividends on its net foreign asset position—declined sharply during the interwar period, while the U.S. income balance rose significantly (Panel B). This provides compelling evidence that the military hegemon has historically earned an excess return on its international investment position—a phenomenon commonly referred to as the "exorbitant privilege".

**Figure 2** Net and Gross U.S. FDI Positions (% of GDP, 1950-2015)




Source: U.S. BEA from 1950 to 1969, and Lane and Milesi-Ferretti (2018) from 1970 onward.

A significant source of this exorbitant privilege can be attributed to the concentration of the hegemon's foreign assets in equities and foreign direct investments (FDIs). Before 1914, the United Kingdom was the world's principal exporter of capital. British foreign outward investment stock (portfolio equity and FDI) was approximately five times larger than that of the United States. London financed infrastructure, utilities, and sovereign borrowing across the globe, to the tune of 130% of U.K. GDP (Wilkins (1970), Table X.1, page 201). By

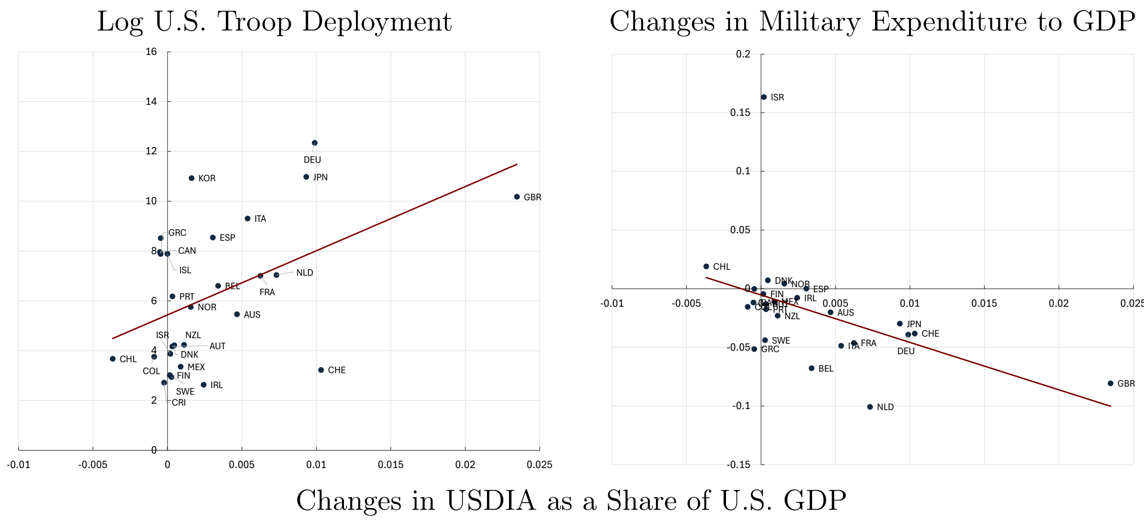
1970, however, U.S. outward FDI had grown to three times the size of the U.K.'s, reflecting the rise of U.S. multinational firms (Lane and Milesi-Ferretti (2018)).[1] Additionally, unlike its overall Net International Investment Position (NIIP), the U.S. Net FDI position has remained consistently positive between 1950 and 2015, with the exception of a brief dip into negative territory in 2000–2001 (Figure 2). For example, in 1991, at the end of the Cold War, the U.S. IIP excluding gold and official reserves stood at approximately \$400 billion in deficit, consisting of about \$160 billion in net FDI and \$550 billion in net portfolio debt. Other investment categories, including net portfolio equity, recorded small negative positions relative to the overall IIP. Indeed, Curcuru, Dvorak and Warnock (2008) estimate that the U.S. cross-border returns differentials are largely explained by a composition effect, which is the concentration of U.S. foreign assets in equities and FDI.

The U.S. also bears a disproportionate share of the collective defense burden, particularly in countries where it has significant FDIs. Figure 3 illustrates this relationship by plotting the annual change in U.S. direct investment abroad as a percentage of U.S. GDP against two key indicators: the change in U.S. troop deployment (Panel A) and the annual change in military expenditure as a percentage of the host country's GDP (Panel B), averaged over the period from 1950 to 1990. The figure points to two clear patterns: countries experiencing faster increases in U.S. FDI inflows are also those where the U.S. deployed more troops, and these countries simultaneously experienced larger declines in their military spending. This evidence underscores the link between U.S. economic and military commitments abroad, as well as the free-riding behavior of allied nations.

In this paper, we present a framework that connects these three stylized facts. We propose a theory explaining how a vulnerable ally can leverage financial integration with a hegemonic power to secure military protection in the face of external threats. This is achieved by subsidizing foreign direct investments (FDIs) within the ally's borders and encouraging

[1] On another yardstick, in 1945, total U.S. investment abroad (government lending plus portfolio and direct investments) was almost twice the total foreign investment in the United States (Wilkins (2004), Table 10.1, page 564).

**Figure 3** Skin in the Game, Military Protection, and Free Riding (Average Changes, 1950-1989)




domestic residents to purchase the hegemon's bonds. The resulting security guarantee is reflected in an asymmetric distribution of defense burdens: the hegemon assumes a disproportionately larger share, allowing the ally to reduce its military expenditures. In return, the hegemon benefits by earning an excess return on its international investment position. This arrangement is stable because it is Pareto-improving, with both the hegemon and the ally benefiting.

The environment consists of a world economy with three countries: a military hegemon referred to as the United States, an Ally, and an external threat referred to as the Enemy. Each country is endowed with Lucas trees that produce a nontradable good and a country-specific tradable good. The United States and its Ally are connected through trade and financial linkages, while the Enemy operates as a closed economy. The United States and the Ally trade shares of their respective tradable trees and a bond denominated in units of the U.S. nontradable good. The model unfolds over three periods: zero, one, and two. In period one, the Enemy may initiate a war by attacking the Ally with a given probability. The outcome of the war depends on the military effort exerted by both the Ally and the hegemon, which can decide how many military resources, if any, to deploy. Decisions regarding military resources and portfolio allocations are made in period zero.

Equipped with this framework, we derive three main results. First, the hegemon intervenes in a war between the Enemy and the Ally if, and only if, it holds shares in the Ally's tradable tree. These holdings create a direct financial stake in the outcome of the conflict ("skin in the game"), incentivizing the United States to act in order to protect its financial interests by influencing the outcome in favor of the Ally. [2] Furthermore, the size of the U.S.'s military intervention at time one decreases with its net foreign asset position and, for a given net position, increases with its gross equity assets and liabilities. Larger gross positions amplify the U.S. portfolio's exposure to the war's outcome—on the asset side, through a greater direct financial stake in the Ally's economy, and on the liability side, through the foreign currency return of U.S. equities.

Second, the international investment position asymmetrically affects the defense incentives of the Ally and the hegemon. If the U.S. intervenes in the war, the Ally's military expenditure increases with the U.S. net foreign asset position but decreases with the gross equity assets and liabilities, for a given net position. Conversely, the U.S.'s military expenditure is increasing in its military effort at time one. This asymmetry suggests that, as financial integration between the two economies deepens, the U.S. assumes a larger share of the defense burden, allowing the Ally to reduce its own military spending and rely more heavily on the U.S.'s implicit commitment to intervene.

Third, the Ally's optimal capital account policy involves subsidizing the issuance of Ally shares to U.S. households and encouraging domestic holdings of U.S. bonds. This result is central to the paper, as it demonstrates how the Ally can strategically distort its international portfolio to secure a military guarantee from the United States. In doing so, it provides a rationale for the stylized facts described earlier. In the decentralized equilibrium, where

[2] Importantly, in our model, financial linkages are the sole channel through which the U.S. is exposed to the conflict's outcome. This is due to the assumption that international trade collapses entirely during a war, regardless of its outcome. Without this assumption, the U.S. would have an additional incentive to deploy military resources to protect trade flows, even in the absence of financial linkages. By abstracting from trade considerations, the model isolates and highlights the role of international portfolio positions in shaping the U.S.'s incentives to intervene. (A more general framework is provided in Online Appendix XX, where trade is disrupted but not entirely shut down.)

portfolio decisions are made independently, Ally households fail to internalize how their portfolio choices influence the outcome of a potential war. This misalignment creates a wedge between private and social incentives, offering a clear justification for government intervention. The optimal policy consists of two key components. First, the Ally subsidizes the issuance of its shares to U.S. households to increase U.S. financial exposure to the conflict, thereby strengthening U.S. incentives to intervene. However, to attract U.S. investors, the Ally must offer a higher expected return on its shares. This excess return compensates U.S. investors for the costs of investing abroad and represents the price the Ally pays to secure the U.S. security guarantee. Second, the Ally seeks to influence its net international investment position by encouraging domestic savings through a subsidy on U.S. bond holdings. This policy incentivizes domestic households to hold more U.S. bonds than they would otherwise choose, given the concurrent incentives to issue Ally shares. By reducing the U.S. net foreign asset position, the Ally reinforces the U.S.'s incentive to protect its foreign investments in the event of a conflict. However, this policy comes with a cost: it distorts the intertemporal allocation of consumption.

The Ally's optimal capital account policy has broader economic implications that extend beyond securing a military guarantee. Compared to the decentralized equilibrium, this policy leads to an appreciation of the dollar and a shift in the U.S. trade balance into deficit. Additionally, the Ally's military expenditure decreases, while U.S. military expenditure increases. Despite this asymmetry, both countries ultimately benefit. The Ally gains from the implicit security guarantee provided by financial integration with the U.S., while the United States benefits from the "exorbitant privilege" associated with providing this security guarantee.

Finally, we turn to the data to examine the mechanism at the core of the model: the relationship between the U.S.'s financial stake in its allies and its provision of security. Our analysis shows that bilateral FDI positions predict U.S. troop deployments abroad, with this relationship being particularly pronounced among non-NATO countries, where no formal security commitments exist to substitute for the financial channel. Additionally, we find that countries receiving higher levels of U.S. direct investment tend to spend less on their own defense, although this result weakens once common time trends are accounted for. These findings are based on a small panel of countries with long bilateral FDI histories from the BEA and should be viewed as promising preliminary evidence. A more robust assessment will require a larger sample and a research design capable of identifying causal effects, both of which are currently in progress.

**Related Literature** Our paper relates to the literature along multiple dimensions. First, a growing body of research examines how military power and geopolitical competition shape international financial arrangements. Eichengreen, Mehl and Chiţu (2019) document that countries relying on the United States for their security hold a disproportionately larger share of foreign reserves in U.S. dollars. Pflueger and Yared (2024) develop a model in which military spending, geopolitical risk, and government bond prices are jointly determined, showing that hegemonic powers enjoy a funding advantage that rises with geopolitical tensions; their analysis focuses on bond markets and treats military hegemony as exogenous. Kim (2025) builds a general-equilibrium model in which the U.S. security umbrella—taken as an exogenous global public good—lowers geopolitical risk, generates a security-based convenience yield on dollar assets, and rationalizes persistent U.S. trade deficits. Clayton, Maggiori and Schreger (2026) examine how global imbalances translate into *power* imbalances, showing that a large foreign creditor's threat to dump U.S. bonds can extract geopolitical concessions when debt levels are high enough to create rollover risk—as illustrated by the 1956 Suez crisis. Our paper differs from the others in two critical respects. First, we *endogenize* the hegemon's military intervention as a function of its financial exposure: the ally's portfolio choice creates the incentives to intervene, so causality runs from FDI to military protection. Second, the relevant financial instrument is equity (e.g., FDI), not bonds or reserves—it is the hegemon's "skin in the game" through direct investment that triggers protection, generating predictions for portfolio composition that are absent from bond-market models.

The paper also adds a layer to the large and well-established literature on the U.S. exorbitant privilege. In particular, Gourinchas and Rey (2007) documents that the United States earns an excess return on its gross foreign assets relative to what it pays on its liabilities to foreigners. They attribute this differential to two factors: a return discount (also called a convenience yield in other parts of this literature), whereby foreigners earn lower returns on their U.S. holdings than Americans earn on comparable assets abroad, and a composition effect, whereby the U.S. holds risky, high-return assets abroad—primarily equity and direct investment—while financing itself through safe, low-return liabilities such as Treasury bonds. Curcuru et al. (2008) later found that the within-class differential on portfolio securities is smaller than previously estimated, while the differential is driven by the composition tilt toward equity and FDI and a persistent income yield advantage on U.S. direct investment abroad as stressed in Figure 1. Our provides a structural explanation for both the composition effect and the FDI income differential. Our theory predicts that U.S. military protection lowers war risk in host countries, encouraging U.S. firms to make direct investments abroad while attracting foreign capital into safe U.S. assets. This security-driven mechanism generates the portfolio asymmetry that both papers document. Empirically, we focus on FDI positions as the driver of the exorbitant privilege tied to the U.S. security umbrella.

A long-standing tradition linked foreign direct investment and security. The relationship between FDI and military presence has traditionally been studied primarily as "flag leads trade": security commitments facilitate investment by reducing political risk. Biglaiser and DeRouen (2007) document that U.S. FDI flows to countries hosting U.S. troop deployments, a pattern specific to U.S. investors. More recently, Aiyar, Malacrino and Presbitero (2024) show that geopolitical alignment increasingly shapes bilateral FDI, with "friend-shoring" becoming more salient since 2018. While Alfaro and Chor (2024) document that U.S.–China tensions have reallocated FDI toward countries geopolitically aligned with the United States. Frieden (1994) provides an early analysis of how different types of cross-border investments attract different forms of political and military protection: site-specific investments with easily appropriable rents historically invited direct control (including colonialism), while portfolio and manufacturing investments relied on cooperative enforcement. Brooks (2005) extends this logic to modern production networks, arguing that multinational firms create vested interests in stability, though without modeling portfolio incentives or deriving asset-pricing implications. We explore causation in the opposite direction from FDIs to security.

Third, the paper relates to the well-established literature on security burden sharing and free riding. Olson and Zeckhauser (1966) develop a theory of alliances, assuming that defense is a public good and that larger members disproportionately shoulder alliance costs and implying that security guarantees should induce lower military expenses. Nordhaus, Oneal and Russett (2012) examines how external security shapes national defense spending. They find that a one-percentage-point increase in the likelihood of a militarized dispute raises defense spending by about 3 percent. However, Allen, VanDusky-Allen and Flynn (2016) find that U.S. troop deployments influence host-state defense spending, with allies increasing their defense burden while non-allies tend to reduce it. Empirically, we find that faster growth in U.S. and foreign direct investments predicts lower defense expenditure, providing an additional economic rather than strategic reason why countries might free ride on the U.S. security guarantee.

A large body of finance literature focused on war risk, the equity premium, and portfolio choice. Pástor and Veronesi (2013) develop a general equilibrium model where political uncertainty commands a conditional risk premium that is stronger when macroeconomic conditions are poor. Cortes, Vossmeyer and Weidenmier (2022) find that higher defense spending stabilizes aggregate and sectoral volatility by securing demand for defense-related output, reducing uncertainty without directly altering the equity premium. Hirshleifer, Mai and Pukthuanthong (2025a) construct a war discourse index from 160 years of news data and show that increases in war-related news predict higher future excess returns, consistent with a priced war-risk premium. Hirshleifer, Mai and Pukthuanthong (2025b) shows that a war-related factor extracted from news explains the cross-section of stock returns, indicating that exposure to war risk is systematically priced. On the portfolio side, Jiao and Ng (2025) show that institutional investors overweight equities in allied countries with high defense spending while underweighting rivals undertaking similar military buildups. Converse and Mallucci (2025) document that rising geopolitical risk leads bond fund managers to concentrate portfolios toward geopolitically aligned countries. Crosignani, Han and Macchiavelli (2025) provide micro evidence that U.S. export controls—a tool of geoeconomic competition—reduce the returns and increase the volatility of exposed mutual fund portfolios. We study war risk in an open-economy, general equilibrium setting and derive implications for both equity returns and country portfolios. Our paper differs from this literature in a fundamental respect: we *endogenize* the hegemon's military intervention as a function of its financial exposure. Rather than treating U.S. security provision as exogenous or studying how geopolitical risk affects portfolios, we show that the ally's portfolio choice *creates* the hegemon's incentive to intervene. In our analysis, the direction of causality runs from FDI positions to military protection, not the reverse.

Finally, the idea that commerce promotes peace traces back to Montesquieu, Kant, and Mill.[3] Martin, Mayer and Thoenig (2008) provides a nuanced theoretical and empirical treatment: bilateral trade reduces the probability of conflict between trading partners, but multilateral openness can increase it by lowering bilateral dependence. Huang, Li and Meissner (2025) use technological advances in air transport as an instrument for trade flows and find that trade causally reduces interstate military conflict. Our paper extends this classical logic from trade to financial integration: cross-border FDI positions create "skin in the game" that makes military protection incentive-compatible for the hegemon.

The rest of the paper is organized as follows. Section 2 develops the model and discusses its solution. It also presents the paper's main theoretical results. Section 3 provides evi-

[3]Montesquieu, *The Spirit of Laws* (1748): "Peace is the natural effect of trade." Kant, *Perpetual Peace* (1795): "The spirit of commerce, which is incompatible with war, sooner or later gains the upper hand in every state." Mill, *Principles of Political Economy* (1848): "It is commerce which is rapidly rendering war obsolete, by strengthening and multiplying the personal interests which are in natural opposition to it."

dence on the model implications. Section 4 concludes. An Online Appendix (referred to as "Appendix" for brevity throughout the paper) provides additional details.

# 2 Model

In this section, we describe the model and discuss its properties. We begin by describing the environment and the structure of the two economies, including preferences, asset menu, and market structures. We then introduce war, model its effects, and incorporate military spending into the framework to draw testable implications in the data. The model's empirical implications are discussed and tested in the following section.

## 2.1 Setup

The world economy is populated by three countries: a military Hegemon, denoted by U for the United States, an Ally, denoted by A, and an Enemy, denoted by E.[4] Each country is populated by a unit measure of homogeneous households and is endowed with Lucas trees that produce a nontradable good and a country-specific tradable good. We focus on the interaction between the U.S. and the Ally. For simplicity, we assume that only the U.S. and the Ally have trade and financial linkages. The Enemy is effectively a closed economy.

Time is discrete, and there are three periods, $t = 0, 1, 2$. At time $t = 1$, the Enemy may attack the Ally, triggering a war between the two countries. War affects international trade and disrupts supply and demand in both the U.S. and the Ally. The outcome of the war, modeled as the degree of disruption in the Ally, depends on the Ally's military decisions and capabilities. The Hegemon can intervene to improve the outcome of the war for the Ally. The Hegemon also chooses the strength of the intervention.

[4] In the rest of the paper, we use U and U.S. interchangeably.

### 2.1.1 The Hegemon

Households in the U.S. maximize

$$U_0 = \mathbb{E}_0 \sum_{t=0}^{2} \beta^t \mu_t \log c_t, \tag{1}$$

where $c_t$ is a consumption basket including the domestic nontradable good, $c_{NT,t}$, the U.S. tradable good, $c_{U,t}$, and the ally tradable good, $c_{A,t}$, given by $c_t \equiv \left[ (c_{NT,t})^{\omega_t} (c_{U,t})^{\alpha_t} (c_{A,t})^{\gamma_t} \right]^{\frac{1}{\mu_t}}$. The preference parameters $\{\omega_t, \alpha_t, \gamma_t\}$ are nonnegative and potentially stochastic, with $\mu_t \equiv \omega_t + \alpha_t + \gamma_t$. We assume that the nontradable good is the numeraire in the U.S. and normalize its price to be 1 in domestic currency. Hence, $p_{NT,t} = 1$ for $t = 0, 1, 2$.

In each period, households receive an endowment of the domestic nontradable and tradable goods, denoted by $y_{NT,t}$ and $y_{U,t}$ respectively. They consume and choose a portfolio of assets to bring to the next period. Households can borrow or save in a one-period international bond, denoted by $b_t$, that pays one unit of the U.S. nontradable good.

They can also invest in shares of the Ally's tradable tree, denoted by $s_{A,t+1}$, or issue shares of their own tradable tree, denoted by $-s_{U,t+1}$. The U.S. households' budget constraint is

$$\begin{aligned} &p_t c_t + q_t b_{t+1} + q_{A,t} s_{A,t+1} + q_{U,t} s_{U,t+1} \\ &\quad = y_{NT,t} + p_{U,t} y_{U,t} + b_t + s_{A,t} \left( p_{A,t} y_{A,t} + q_{A,t} \right) + s_{U,t} \left( p_{U,t} y_{U,t} + q_{U,t} \right) - z_t, \end{aligned} \tag{2}$$

where $p_t c_t \equiv c_{NT,t} + p_{U,t} c_{U,t} + p_{A,t} c_{A,t}$ denotes total consumption expenditure, $p_{U,t}$ and $p_{A,t}$ are the dollar prices of the U.S. and Ally tradables, and $q_{U,t}$ and $q_{A,t}$ the dollar prices of their trees. The dollar price of the international bond is $q_t$, while $z_t$ denotes a lump-sum component of U.S. households income, which includes taxes and other transfers.

U.S. households choose consumption components $\{c_{NT,t}, c_{U,t}, c_{A,t}\}_{t=0,1,2}$ and their portfolios, $\{b_t, s_{A,t}, s_{U,t}\}_{t=1,2}$, by maximizing (1) subject to (2). The households' optimization problem can be decomposed into two parts: a static, intratemporal allocation problem—

in which, given total consumption expenditure in the period, households allocate spending across goods; and a dynamic, intertemporal problem—in which households choose their saving and borrowing strategies across various assets over time.

The first-order conditions of the static consumption expenditure allocation problem are

$$\frac{\omega_t}{c_{NT,t}} = \frac{\mu_t}{p_t c_t}, \quad \frac{\alpha_t}{p_{U,t} c_{U,t}} = \frac{\mu_t}{p_t c_t}, \quad \frac{\gamma_t}{p_{A,t} c_{A,t}} = \frac{\mu_t}{p_t c_t},$$

where $\frac{\mu_t}{p_t c_t}$ is the Lagrange multiplier associated with (2) which, in equilibrium, is equal to the marginal utility of consumption expenditures. For simplicity, following **?**, in our analysis, we assume that the endowment process for nontradable goods follows $y_{NT,t} = \omega_t$. This assumption, combined with the market clearing condition $y_{NT,t} = c_{NT,t}$, implies that consumption expenditures are $p_t c_t = \mu_t$, and U.S. households' demands for U.S. and Ally tradables goods take the simple form $p_{U,t} c_{U,t} = \alpha_t$ and $p_{A,t} c_{A,t} = \gamma_t$. Thus, $\gamma_t$ is the dollar value of U.S. imports from the Ally.

The intertemporal consumption-savings decision yields the Euler equations

$$\begin{aligned}
q_t &= \beta \mathbb{E}_t \left[ \Lambda_{t|t+1} \frac{p_t}{p_{t+1}} \right], \\
q_{U,t} &= \beta \mathbb{E}_t \left[ \Lambda_{t|t+1} \frac{p_t}{p_{t+1}} \left( p_{U,t+1} y_{U,t+1} + q_{U,t+1} \right) \right], \\
q_{A,t} &= \beta \mathbb{E}_t \left[ \Lambda_{t|t+1} \frac{p_t}{p_{t+1}} \left( p_{A,t+1} y_{A,t+1} + q_{A,t+1} \right) \right],
\end{aligned}$$

where $\Lambda_{t|t+1} = \frac{\mu_{t+1}/c_{t+1}}{\mu_t/c_t}$ is the U.S. households' stochastic discount. Our assumption for the nontradable endowment process implies that $\Lambda_{t|t+1} \frac{p_t}{p_{t+1}} = 1$, rendering risk-averse households effectively risk-neutral with respect to claims on nontradable goods. While this assumption is admittedly strong, it simplifies the analysis by neutralizing variations in households' marginal utility that are not key for the economic insights of the paper. This happens because any variation in marginal utility is offset by a proportional change in the price of the consumption basket. For example, an unexpected windfall worth one unit of nontradable goods raises consumption (lowering its marginal utility) and simultaneously lowers its price (raising the purchasing power of that unit). These effects exactly offset each other in equilibrium, keeping households' marginal utility of nontradable goods constant. Thus, the asset pricing conditions simplify to

$$\begin{aligned} q_t &= \beta, \\ q_{U,t} &= \beta \mathbb{E}_t \left[ p_{U,t+1} y_{U,t+1} + q_{U,t+1} \right], \\ q_{A,t} &= \beta \mathbb{E}_t \left[ p_{A,t+1} y_{A,t+1} + q_{A,t+1} \right]. \end{aligned} \tag{3}$$

It is important to emphasize that this assumption is made purely for analytical convenience and is not critical to the results of the paper. For completeness, more general results that relax this assumption are provided in Online Appendix XX.

#### 2.1.2 The Ally

Household preferences and budget constraint in the Ally are the mirror image of those in the Hegemon. Ally's households maximize

$$U_0^* = \mathbb{E}_0 \sum_{t=0}^{2} \beta^t \mu_t \log c_t^*$$

where $c_t^* \equiv \left[ \left( c_{NT,t}^* \right)^{\omega_t^*} \left( c_{A,t}^* \right)^{\alpha_t^*} \left( c_{U,t}^* \right)^{\gamma_t^*} \right]^{\frac{1}{\mu_t^*}}$, with $\mu_t^* \equiv \omega_t^* + \alpha_t^* + \gamma_t^*$. The nontradable good is the numeraire also in the Ally, and we normalize its price to be 1 in domestic currency. That is, $p_{NT,t}^* = 1$ for $t = 0, 1, 2$. The exchange rate, denoted by $e_t$, is defined as the price of U.S. nontradables in terms of Ally nontradables, such that an increase in $e_t$ is an appreciation of the dollar. In the absence of a nominal side to the model, we intentionally abuse the word exchange rate to mean the real exchange rate. Similarly, we abuse the word dollar to mean a claim to the numeraire of the U.S.

The Ally households' budget constraint is

$$
\begin{aligned}
& p_t^* c_t^* + q_t^* b_{t+1}^* + q_{U,t}^* s_{U,t+1}^* + q_{A,t}^* s_{A,t+1}^* \\
& \quad = y_{NT,t}^* + p_{A,t}^* y_{A,t} + e_t b_t^* + s_{U,t}^* \left( p_{U,t}^* y_{U,t} + q_{U,t}^* \right) + s_{A,t}^* \left( p_{A,t}^* y_{A,t} + q_{A,t}^* \right) - z_t^*,
\end{aligned}
$$

where $p_t^* c_t^* \equiv c_{NT,t}^* + p_{A,t}^* c_{A,t}^* + p_{U,t}^* c_{U,t}^*$ and all prices are expressed in local currency. As for the U.S., we assume that the endowment process for the Ally nontradable good follows $y_{NT,t}^* = \omega_t^*$, which yields

$$
p_{U,t}^* c_{U,t}^* = \gamma_t^* \quad \text{and} \quad p_{A,t}^* c_{A,t}^* = \alpha_t^*,
$$

where $\alpha_t^*$ is the Ally households' demand for domestic tradable goods and $\gamma_t^*$ is the local currency value of the Ally's imports from the U.S. The asset pricing equations derived from the Ally households' intertemporal problem are

$$
\begin{aligned}
q_t^* &= \beta \mathbb{E}_t \left[ e_{t+1} \right], && (4) \\
q_{U,t}^* &= \beta \mathbb{E}_t \left[ p_{U,t+1}^* y_{U,t+1} + q_{U,t+1}^* \right], \\
q_{A,t}^* &= \beta \mathbb{E}_t \left[ p_{A,t+1}^* y_{A,t+1} + q_{A,t+1}^* \right].
\end{aligned}
$$

Again, as in the case of U.S. households, Ally households are risk-neutral with respect to claims denominated in domestic nontradable goods.

### 2.1.3 Trade and Financial Linkages and Market Clearing

We conclude this section by describing trade and financial linkages between the two countries. The U.S. and Ally households can exchange tradable goods in international markets in which the law of one price holds. Hence, $p_{U,t}^* = e_t p_{U,t}$ and $p_{A,t} = \frac{p_{A,t}^*}{e_t}$. Markets clearing require

$$
p_{U,t} y_{U,t} = \alpha_t + \frac{\gamma_t^*}{e_t} \quad \text{and} \quad p_{A,t}^* y_{A,t} = \alpha_t^* + e_t \gamma_t.
$$

Regarding the asset markets, we assume that international bond trading is frictionless, while trading in stocks is subject to some costs. The first assumption implies that the price of the bond in the Ally currency is $q_t^* = e_t q_t$. This, and the fact that the real interest rates in the two countries are equals, and equal to the inverse of their time discount factor $\beta$, implies that, in the competitive equilibrium of the model, the exchange rate follows a random walk. Using (3) and (4) yields $e_t = \mathbb{E}_t [e_{t+1}]$. Unlike bonds, buying and selling shares of the foreign tree is subject to an iceberg cost that is proportional to the changes in aggregate holdings. This implies that the prices of the Ally tree in the U.S. and the U.S. tree in the Ally are

$$q_{A,t} = \frac{q_{A,t}^*}{e_t} (1 + \psi \Delta \bar{s}_{A,t+1})$$
$$q_{U,t}^* = e_t q_{U,t} \left(1 + \psi^* \Delta \bar{s}_{U,t+1}^*\right)$$

with $\psi, \psi^* > 0$, where $\Delta \bar{s}_{A,t+1} \equiv \bar{s}_{A,t+1} - \bar{s}_{A,t}$ denote changes in U.S. holdings of Ally shares and $\Delta \bar{s}_{U,t+1}^* \equiv \bar{s}_{U,t+1}^* - \bar{s}_{U,t}^*$ denote changes in Ally holdings of U.S. shares. These assumptions reflect the relatively higher liquidity and lower transaction costs on short-term bonds and the higher informational and institutional, and regulatory frictions associated with equities. Markets clearing in the international assets markets require $0 = b_t + b_t^*$, $0 = s_{U,t} + s_{U,t}^*$, and $0 = s_{A,t} + s_{A,t}^*$.

## 2.2 War, military efforts and military expenditures

In this section, we introduce the concept of war and the military dynamics within the model. At $t = 2$, the world is at peace. However, at $t = 1$, there is a probability $\rho \in (0, 1)$ that war could break out between the Enemy and the Ally. The outcome of this war, which influences the extent of economic disruption in the Ally, is determined by the military efforts of both the Ally and the U.S. At $t = 0$, the U.S. and the Ally make strategic decisions regarding their respective military expenditures in preparation for potential conflict.

We model the disruption caused by war through two distinct channels. The first channel operates through the disruption of trade flows. We assume that during a war, non-military trade in goods between the U.S. and the Ally collapses entirely:

$$\gamma_1^* (war) = \gamma_1 (war) = 0.$$

This assumption reflects the physical disruption of supply chains caused by conflicts. For instance, the destruction of critical infrastructure—such as ports, railways, highways, and airports—significantly hinders the international exchange of goods. By assuming that trade collapses completely, the impact of war on U.S. households' utility is transmitted solely through its effect on their financial wealth. This simplification allows us to focus on how international portfolio positions shape the U.S.'s incentives to influence the outcome of the war. It is important to note that the assumption of a complete trade collapse is made purely for expositional purposes. It simplifies the algebra and facilitates the derivation of closed-form results. For completeness, a more general framework is provided in Online Appendix XX, where trade is disrupted but not entirely shut down.

The second channel through which war affects equilibrium is its impact on the Ally's economy. We model this as a proportional negative shock to both the aggregate demand and supply of the Ally's goods. Specifically, we assume that war reduces the aggregate demand and supply of both tradable and nontradable goods by a fraction $1 - \theta_1$, where $\theta_1 \in [\theta, 1)$ represents the outcome of the war. A higher $\theta_1$ corresponds to a more positive outcome for the Ally, with less severe economic consequences. Formally, this assumption has the following implications:

$$\begin{aligned} y_{NT,1}^* (war) &= \theta_1 y_{NT,1}^* (peace) \quad \text{and} \quad \omega_1^* (war) = \theta_1 \omega_1^* (peace), \\ y_{A,1} (war) &= \theta_1 y_{A,1} (peace) \quad \text{and} \quad \alpha_1^* (war) = \theta_1 \alpha_1^* (peace). \end{aligned}$$

These equations capture the direct economic consequences of war on the Ally's production capacity and the diminished consumer spending, resulting in weaker economic activity. Once again, the assumption that war affects the demand and supply of tradable and nontradable goods in the same way is made purely to simplify the notation and keep the model tractable. This simplification does not alter the core insights of the analysis. In the more general framework presented in Online Appendix XX, we relax this assumption and allow for asymmetric effects of war on demand and supply and/or on tradable versus nontradable goods.

The outcome of the war, $\theta_1$, is endogenous and is determined by the military efforts of both the Ally and the U.S. We use the term military effort in a broad sense to refer to all the activities performed by a country to achieve specific military objectives. It encompasses activities with different intensity levels, which can range from routine patrols or training exercises, to providing resources or equipment to a combatant country, to a full scale military intervention. We model military effort as the level of resources that a country allocates to military activities. Formally, denoting with $m_1 \geq 0$ and $m_1^* \geq 0$ the Ally and U.S. military efforts, respectively, we assume that the outcome of the war is determined as:

$$\theta_1 = f\left(m_1, m_1^*\right) \tag{5}$$

where $f : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to [\underline{\theta}, 1)$ is increasing in both arguments, the parameter $\underline{\theta} \geq 0$ represents the worst possible outcome of the war, and the upper limit 1 represents the best possible outcome.[5] In what follows, we assume the following functional form for $f$:

$$f\left(m_1, m_1^*\right) = \underline{\theta} + \left(1 - \underline{\theta}\right) \frac{1 - e^{-\kappa m_1 - \kappa^* m_1^*}}{1 + e^{-\kappa m_1 - \kappa^* m_1^*}},$$

where the parameters $\kappa \geq 0$ and $\kappa^* \geq 0$ represent the effectiveness of the U.S.'s and the Ally's military efforts, respectively. The functional form has several important properties that provide flexibility and realism in analyzing the impact of military efforts on a war outcome. First, it accounts for differences in the quality or efficiency of the military resources

[5] We assume that $\underline{\theta} > \tilde{\theta}$, where $\tilde{\theta}$ solves $\tilde{\theta}\left(y_{NT}^*\right)^{\frac{\omega^*}{\omega^*+\alpha^*}}\left(y_A\right)^{\frac{\alpha^*}{\omega^*+\alpha^*}} = e^{\frac{\beta}{\tilde{\theta}+\beta}\frac{\alpha^*+\gamma^*}{\alpha^*+\omega^*}-1}$, to ensure that the utility of the Ally's households is strictly increasing in $\theta_1$. See the proof of Lemma 1 for further details.

deployed by the U.S. and the Ally. For instance, $\kappa > \kappa^*$ implies that the U.S.'s military effort has a greater impact on the war outcome than the Ally's military effort. This captures asymmetries in military capabilities between the two countries. Second, the functional form features a nonlinear relationship between military efforts and the war outcome. Specifically, the exponential terms ensure diminishing marginal returns to military efforts. As $m_1$ or $m_1^*$ increase, their incremental impact on $\theta_1$ diminishes, reflecting the fact that additional military resources become less effective at higher levels of deployment. Third, the functional form features strategic substitutability between the military efforts of the U.S. and the Ally. Specifically, as $m_1$ increases, the marginal benefit of $m_1^*$ decreases, and vice versa. This means that an increase in one country's military effort reduces the relative effectiveness of the other country's effort in improving the war outcome.

Military effort is costly because it depletes the stock of military resources (e.g., the size of its army, equipment, or other military assets) of the country. Let $a_1 \geq 0$ represent the stock of military resources available to the U.S. at the beginning of time 1 (e.g., the size of its army, equipment, or other military assets). Each unit of military effort $m_1$ requires $h(m_1)$ units of military resources, where $h : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is an increasing and strictly convex function with $h(0) = \frac{\partial h(0)}{\partial m_1} = 0$. Therefore, $a_1$ represents the maximum military effort that the U.S. can exert at time 1, meaning $m_1 \in [0, a_1]$. To produce $a_1$ military resources, the U.S. must invest $x(a_1)$ units of its tradable goods at time 0, where $x : \mathbb{R}_+ \to \mathbb{R}_+$ is an increasing and strictly convex function with $x(0) = \frac{\partial x(0)}{\partial a_1} = 0$. Military expenditures $p_{U,0}x(a_1)$ are financed through lump-sum taxes, therefore they do not impact the budget constraint of U.S. households. At time 2, unused military resources $a_2 = a_1 - m_1$ can be transformed into $\delta a_2$ tradable goods and consumed, where $\delta \in [0, 1]$. Similarly, at time 0 the Ally produces $a_1^*$ military resources by investing $x^*(a_1^*)$ units of its tradable goods, where $x^*$ is increasing and strictly convex with $x^*(0) = \frac{\partial x^*(0)}{\partial a_1^*} = 0$. At time 1, the Ally chooses its optimal military effort $m_1^* \in [0, a_1^*]$, and at time 2 it transforms unused military resources into $\delta^* a_2^*$ units of tradable goods and consume them. In what follows, we operate under the reasonable assumption that, in the event of war, the Ally exerts its maximum military effort ($m_1^* = a_1^*$), whereas the U.S. does not ($m_1 < a_1$). Lemma 1 in the appendix establishes the existence of parameters $\delta$ and $\delta^*$ that ensure this condition holds.

In each country, military decisions—both expenditures at time 0 and effort at time 1—are made by a centralized authority that maximizes the utility of domestic households. This is in contrast with consumption and savings decisions which are made by atomistic households.

## 2.3 Decentralized equilibrium

In this section, we solve for the decentralized equilibrium of the model by working backward, starting from time 2. At time 2, all countries are at peace, and there are no shocks. We drop the time index on variables to denote their peacetime realizations. For example, $y_U$ and $y_A$ represent the peacetime supply of U.S. and Ally tradable goods, respectively. Similarly, $\alpha$ and $\gamma$ ($\alpha^*$ and $\gamma^*$) denote U.S. (Ally) households' demands for domestic and foreign goods, respectively. To streamline the algebra, and focus on the key economic mechanism of the model, in the main body of the paper we assume $\beta = \alpha = \alpha^* = \gamma = \gamma^* = \delta = 1$ and $\delta^* = \omega = \omega^* \downarrow 0$. The propositions at the end of each subsection generalize our results for a generic set of parameters.

**Equilibrium at t=2** Let $w_2 = b_2 + s_{A,2} p_{A,2} y_A + s_{U,2} p_{U,2} y_U$ represent U.S. financial wealth—i.e., the return on its portfolio of international assets—at the beginning of time 2. Since time 2 is the final period, U.S. households consume their accumulated wealth entirely, with no savings carried forward. Combining their budget constraint with the goods market clearing conditions, $p_{A,2} y_A = \frac{1}{e_2} + 1$ and $p_{U,2} y_U = 1 + \frac{1}{e_2}$, yields the value of the exchange rate that clears the markets:

$$e_2 = \frac{1}{1 - w_2}. \tag{6}$$

As common in this class of models, an increase in U.S. wealth leads to a stronger dollar.

**Equilibrium at t=1** Between time 1 and time 2, U.S. wealth evolves according to the law of motion:

$$w_2 = \frac{\gamma_1^*}{e_1} - \gamma_1 + w_1, \tag{7}$$

where $\frac{\gamma_1^*}{e_1} - \gamma_1$ is the time-1 U.S. trade balance, denominated in dollars.[6] Using the goods markets clearing conditions, $p_{A,1}y_{A,1} = \frac{\alpha_1^*}{e_1} + \gamma_1$ and $p_{U,1}y_{U,1} = \alpha_1 + \frac{\gamma_1^*}{e_1}$, and the asset pricing equations, $q_{A,1} = \frac{1}{e_2} + 1$ and $q_{U,1} = 1 + \frac{1}{e_2}$, we can express $w_1$ as a function of the time-1 and time-2 exchange rate, as follows:

$$w_1 = b_1 + s_{A,1}\left(\frac{\alpha_1^*}{e_1} + \gamma_1 + \frac{1}{e_2} + 1\right) - s_{U,1}^*\left(1 + \frac{\gamma_1^*}{e_1} + 1 + \frac{1}{e_2}\right). \tag{8}$$

Recall now that the exchange rate follows a random walk and, therefore, it is constant between time 1 and time 2, so that $e_1 = e_2$. Armed with this, we combine (6), (7), and (8) to solve for $w_1$ as a function of international portfolios $\left\{b_1, s_{A,1}, s_{U,1}^*\right\}$:

$$w_1 = \frac{b_1 + s_{A,1}\left(\frac{\alpha_1^*+1}{\gamma_1^*+1} + 1\right)(\gamma_1 + 1) - s_{U,1}^*(3 + \gamma_1)}{1 + s_{A,1}\frac{\alpha_1^*+1}{\gamma_1^*+1} - s_{U,1}^*}, \tag{9}$$

The exchange rate that clears the markets at time 1 is then given by: $e_1 = \frac{\gamma_1^*+1}{\gamma_1+1-w_1}$. In the event of war, $\gamma_1(war) = \gamma_1^*(war) = 0$, and $\alpha_1^*(war) = \theta_1$. Therefore, the gross return on the U.S. portfolio of international assets is:

$$w_1(war) = \frac{b_1 + (2 + \theta_1)s_{A,1} - 3s_{U,1}^*}{1 + (1 + \theta_1)s_{A,1} - s_{U,1}^*}.$$

Armed with this result, we are now ready to analyze the incentives for the U.S. to

[6] Since there are no shocks at time 2, all assets are risk-free and and yield the same rate of return. This also implies that households do not adjust their holdings of foreign shares due to the portfolio adjustment costs $\psi$ and $\psi^*$, maintaining their positions unchanged: $s_{U,2}^* = s_{U,1}^*$ and $s_{A,2} = s_{A,1}$. The assumption of no shocks at time 2 simplifies the analysis of the model, as it eliminates the need to account for the portfolio rebalancing in the final period. The assumption can be made without any loss of generality because the main goal of the analysis is to investigate the determinants of international portfolio positions at $t = 0$, before the possible onset of war, rather than the portfolio's evolution or adjustments in subsequent periods.

intervene in the war. U.S. households' utility in case of war is

$$U_1\left(war\right) = \log\left(1 + a_2\right) - 2\log\left(1 + \frac{1}{e_1\left(war\right)}\right)$$

with $a_2 = a_1 - h\left(m_1\right)$ and $e_1\left(war\right) = \frac{1}{1 - w_1(war)}$. The first order condition with respect to $m_1$ is

$$\underbrace{\frac{s_{A,1}}{1 - s^*_{U,1} + s_{A,1}\left(1 + \theta_1\right)}}_{\frac{\partial e_1/e_1}{\partial \theta_1}} \underbrace{\frac{2}{1 + e_1\left(war\right)}}_{\frac{\partial U_1(war)}{\partial e_1/e_1}} \frac{\partial \theta_1}{\partial m_1} = \frac{1}{1 + a_2} \frac{\partial h\left(m_1\right)}{\partial m_1} \tag{10}$$

The first term on the left-hand side of this equation, which is the semi-elasticity of the exchange rate with respect to $\theta_1$, measures the sensitivity of the U.S. portfolio to the outcome of the conflict. If U.S. wealth is positively correlated with $\theta_1$, then a more favorable war outcome appreciates the exchange rate. The second term, measures the change in utility induced by a 1% appreciation of the dollar. Overall, these two terms capture the marginal benefit of steering the outcome of the war in favor of the Ally, while the right-hand side captures the marginal cost of effort.

It is immediate from (10) that the U.S. is motivated to deploy military resources if and only if it holds shares in the Ally's tradable assets, i.e. $s_{A,1} > 0$. These holdings create a direct financial stake in the conflict's outcome for the U.S. ("skin in the game"), prompting it to act in order to protect its financial interests by influencing the outcome in favor of the Ally. If $s_{A,1} = 0$, then it is optimal for the U.S. not to intervene in the war at all, that is $m_1 = 0$. It is important to emphasize that, in our model, financial linkages are the only channel through which the U.S. is exposed to the conflict's outcome. This is due to our assumption of a complete collapse of international trade during a war, regardless of its outcome. Without this assumption, the U.S. would have an additional incentive to deploy military resources to protect international trade, even in the absence of financial linkages. By abstracting from trade considerations, however, the model allows us to more clearly isolate and analyze the role of international portfolio positions in shaping the U.S.'s incentives to influence the outcome of the conflict.

While the U.S. intervenes in the war if and only if it holds shares in the Ally's tradable tree, the relationship between its holdings and the level of military effort is non-monotonic. On the one hand, a higher $s_{A,1}$ increases the exposure of the U.S. portfolio to the conflict, raising the semi-elasticity of the exchange rate and the marginal benefit of each unit of military effort. On the other hand, a higher $s_{A,1}$ also raises U.S. wealth ex ante; when the U.S. enters the war with greater financial wealth, the marginal benefit of intervening to prevent further losses diminishes. This lowers the marginal utility of additional wealth—the second term on the left-hand side of (10). The relative strength of the two channels depends on the U.S. holdings of the Ally's tradable tree. When $s_{A,1}$ is low, the exposure channel dominates, making optimal U.S. military effort increasing in U.S. holdings of Ally shares. When $s_{A,1}$ is high, the wealth channel may dominate, making optimal U.S. military effort decreasing in those holdings.

The U.S.'s incentive to deploy military resources depends not only on its own exposure to the Ally's economy, but also on the Ally's exposure to the U.S. economy. In particular, conditional on $s_{A,1} > 0$, the U.S.'s optimal military effort is increasing in the Ally's holdings of shares in the U.S.'s tradable tree, $s^*_{U,1}$. A higher $s^*_{U,1}$ not only raises the marginal utility of a stronger dollar, by reducing U.S.'s wealth ex ante, but it also increases its sensitivity to the outcome of the conflict. When the U.S. intervenes and the dollar strengthens, the Ally's expenditure on U.S. goods falls, reducing the return on the U.S.'s tradable tree. If the Ally holds U.S. shares ($s^*_{U,1} > 0$), this lower return reduces the value of U.S. liabilities, which in turn raises U.S. wealth further (see equation (8)). This increases the marginal benefit of the U.S.'s military effort, prompting the U.S. to allocate additional military resources.[7]

These results highlight the distinct roles of *gross* vs *net* international asset positions in

[7]This mechanism, which links asset returns to the U.S.'s military effort, also impacts the return on the Ally's tradable tree. As the dollar strengthens, the dollar-denominated return on the Ally's tradable tree declines. This, in turn, negatively affects U.S. wealth, reducing its sensitivity to $\theta_1$. However, the direct effect of U.S. intervention on the return of the Ally's tradable tree—operating through the conflict's outcome ($\theta_1$)—remains the dominant factor. As a result, the U.S.'s wealth exposure to the outcome of the conflict is increasing in its holdings of the Ally's tradable tree.

shaping the U.S. incentives to intervene. Let $\bar{b}_1 \equiv q_0 b_1 + q_{A,0} s_{A,1} + q_{U,0} s_{U,1}$ denote the net foreign asset position of the U.S. at the end of time 0. Then, U.S. financial wealth at time 1 in the two states of the world, *war* and *peace*, can be expressed as

$$w_1\left(war\right) = \bar{b}_1 + \left(1-\rho\right)\Delta w_1$$

$$w_1\left(peace\right) = \bar{b}_1 - \rho\Delta w_1$$

where $\Delta w_1 \equiv \frac{s^*_{U,1} - s_{A,1}\left(2-\theta_1+\theta_1\bar{b}_1\right)}{1-s^*_{U,1}+s_{A,1}(1+\theta_1-\rho\theta_1)}$ represents the wealth transfer that occurs between the two states of the world.[8] This representation allows us to isolate the impact of the gross assets ($s_{A,1}$) and liabilities ($s^*_{U,1}$) positions of the U.S. in shaping its incentives to deploy military resources, as any change in gross positions is offset by an equivalent increase or decline in the U.S. holding of bonds, such that its net foreign asset position remains constant.

Consistent with the discussion above, the gross positions affects the U.S. incentives to intervene mostly by changing the sensitivity of the U.S. portfolio to the outcome of the conflict (ie, the first term on the right-hand side of equation (10)). A larger gross foreign assets/liabilities position increases the marginal benefit of the intervention by increasing the U.S. exposure to the conflict's outcome. In contrast, the net foreign assets position affects the U.S. incentives to deploy military resources mostly by changing the marginal utility of wealth (ie, the second term on the right-hand side of equation (10)). A higher U.S. net international asset position reduces the marginal benefit to intervene by lowering the marginal utility of wealth. The following proposition summarizes and generalizes these results.

**Proposition 1** (U.S. military effort)**.** *The U.S. intervenes in the war if and only if it holds a positive amount of shares of the Ally's tradable tree. If* $s_{A,1} > 0$*, the level of the U.S. military effort is decreasing in its net foreign asset position, and, for a given net position—increasing in its gross equity assets* ($s_{A,1}$) *and liabilities* ($s^*_{U,1}$) *positions.*

[8]Notice that for an equilibrium to be well defined we must have $\bar{b}_1 < 2-\rho$ and $\Delta w_1 \in \left(-\frac{2-\bar{b}_1}{\rho}, \frac{1-\bar{b}_1}{1-\rho}\right)$.

**Equilibrium at t=0** At time 0, households choose their portfolios of international assets, and the government decides how much to invest in developing military resources. We begin with the government's problem, taking international portfolios as given, because military investment affects the outcome of a potential conflict and thus the returns on assets. We then solve for households' portfolio choices and derive the decentralized equilibrium of the model.

Consider the Ally household's utility, as a function of $a_1^*$:

$$U_0^* = \log\left(1 - x^*\left(a_1^*\right)\right) - 2\log\left(1 + e_0\right) + \rho\left[2\log\left(\theta_1\right) - 2\log\left(1 + e_1\left(war\right)\right)\right],$$

where $e_0 = \mathbb{E}_0\left[e_1\right]$. Now, recall that in the event of a conflict the Ally deploys all its military resources, that is $m_1^* = a_1^*$. The first order condition with respect to $a_1^*$ equates the marginal benefit at time 1 of additional military resources in case of a conflict, to the marginal cost at time 0 of committing those resources: :

$$2\rho\left[\frac{1}{\theta_1} - \frac{s_{A,1}}{1 - s_{U,1}^* + s_{A,1}\left(1 + \theta_1\right)}\left(\frac{e_1\left(war\right)}{1 + e_1\left(war\right)} + \frac{e_1\left(war\right)}{1 + e_0}\right)\right]\frac{\partial\theta_1}{\partial m_1^*} = \frac{1}{1 - x^*\left(a_1^*\right)}\frac{\partial x^*\left(a_1^*\right)}{\partial a_1^*}, \tag{11}$$

The terms in parenthesis on the left-hand side is the marginal benefit for the Ally of improving the outcome of the conflict. It has two parts. The first is a direct stabilization benefit, $1/\theta_1$, which captures the value of mitigating wartime disruptions to tradable and nontradable goods supply. The second is a transfer component that reflects how a higher $\theta_1$ shifts wealth toward the U.S. when the U.S. is financially exposed to the war outcome. From the Ally's perspective this term is negative, as those gains accrue to U.S. households. If $s_{A,1} = 0$, the U.S. bears no exposure and the transfer channel vanishes.

International portfolio positions affect the Ally's incentives to invest in military resources both directly—by changing how war outcomes are shared across countries—and indirectly—by shaping the U.S. incentives to deploy military resources in the event of war. When $s_{A,1} > 0$, larger gross foreign asset/liability positions increase the sensitivity of U.S. wealth to the war outcome. An improvement in $\theta_1$ then induces a larger wealth transfer to the U.S., reducing the Ally's net marginal benefit from improving the outcome of the conflict. In addition, larger gross foreign asset/liability positions increase the U.S. military effort at time 1, improving the expected outcome of the conflict. As $\theta_1$ increases, both the direct marginal benefit of the Ally's military effort, $1/\theta_1$, and its marginal effectiveness, $\partial\theta_1/\partial m_1^*$, decline, further weakening the Ally's incentive to invest in military resources. The next proposition formalizes these results.

**Proposition 2** (Ally military expenditure)**.** *If $s_{A,1} > 0$, the Ally's military expenditure is increasing in the U.S. net foreign asset position, and, for a given net position—decreasing in the gross equity assets and liabilities positions.*

Turning to the U.S., the government chooses $m_1$ at time 1 to equate its marginal benefit and marginal cost. Hence, by the envelope theorem, its decision to invest in military resources does not depend on the marginal benefit of its military effort. However, the level of military effort deployed in a conflict affects the stock of resources available at time 2 and therefore their marginal value. When the U.S. anticipates deploying more resources in the event of war—for example, because financial exposure to the war outcome is higher, as described in Proposition 1—the shadow value of additional military capacity rises, increasing optimal investment at time 0.

**Proposition 3** (U.S. military expenditure)**.** *The U.S.'s military expenditure is increasing in its military effort.*

These comparative statics highlight the asymmetric impact of international positions on defense investment incentives. As gross financial integration between the two economies increases, the U.S.'s incentives to invest in military resources strengthen, while the Ally's incentives weaken. Consequently, the U.S. tends to bear a larger share of the defense burden as financial interdependence deepens, and the Ally reduces its own spending, relying more on the implicit security guarantee provided by financial integration with the U.S.

We are now ready to characterize the decentralized equilibrium of the model. The U.S.'s and the Ally's households' asset-pricing conditions for shares of the Ally's tradable tree are:

$$q^*_{A,0} = \mathbb{E}_0 \left[e_1 \left(\gamma_1 + 1\right) + \alpha^*_1 + 1\right],$$
$$q_{A,0} = \mathbb{E}_0 \left[\frac{e_1 \left(\gamma_1 + 1\right) + \alpha^*_1 + 1}{e_1}\right],$$

where we used $p^*_{A,1} y_{A,1} + q^*_{A,1} = e_1 \left(\gamma_1 + 1\right) + \alpha^*_1 + 1$, while their asset pricing conditions for shares of the U.S.'s tradable tree are

$$q_{U,0} = \mathbb{E}_0 \left[2 + \frac{\gamma^*_1 + 1}{e_1}\right],$$
$$q^*_{U,0} = \mathbb{E}_0 \left[e_1 \left(2 + \frac{\gamma^*_1 + 1}{e_1}\right)\right].$$

where we used $p_{U,1} y_{U,1} + q_{U,1} = 2 + \frac{\gamma^*_1+1}{e_1}$. Using $q_{A,0} = \frac{q^*_{A,0}}{e_0} \left(1 + \psi_A s_{A,1}\right)$, $q^*_{U,0} = e_0 q_{U,0} \left(1 + \psi_U s^*_{U,1}\right)$, and $e_0 = \mathbb{E}_0 \left[e_1\right]$, we can combine the asset pricing equations to obtain two conditions that the equilibrium portfolios $s_{A,1}$ and $s^*_{U,1}$ must satisfy:

$$\frac{1}{1 + \psi_A s_{A,1}} = 1 + \frac{cov\left(e_1, \gamma_1 + 1 + \frac{\alpha^*_1+1}{e_1}\right)}{\mathbb{E}_0\left[e_1\right] \mathbb{E}_0 \left[\gamma_1 + 1 + \frac{\alpha^*_1+1}{e_1}\right]}, \tag{12}$$

$$1 + \psi_U s^*_{U,1} = 1 + \frac{cov\left(e_1, 2 + \frac{\gamma^*_1+1}{e_1}\right)}{\mathbb{E}_0\left[e_1\right] \mathbb{E}_0 \left[2 + \frac{\gamma^*_1+1}{e_1}\right]}. \tag{13}$$

Finally, $b_1$ is determined by the U.S. time-0 budget constraint as follows:

$$b_1 + \frac{q^*_{A,0}}{e_0} s_{A,1} - \frac{q^*_{U,0}}{e_0} s^*_{U,1} = \frac{1}{e_0} - 1. \tag{14}$$

The decentralized portfolio allocation can be obtained with a guess-and-verify approach. Suppose that $b_1 = s_{A,1} = s^*_{U,1} = 0$. Then, the exchange rate at time 1 is constant across states, $e_1\left(war\right) = e_1\left(peace\right) = 1$. It follows that the covariance terms in (12) and (13) are zero, which confirms that $s_{A,1} = s^*_{U,1} = 0$. Since $e_0 = \mathbb{E}_0[e_1] = 1$, equation (14) then yields $b_1 = 0$.

**Proposition 4** (Decentralized equilibrium)**.** *In the decentralized equilibrium the U.S. and the Ally are in financial autarky, that is* $b_1 = s_{A,1} = s^*_{U,1} = 0$.

While stark, this result provides a useful benchmark for the optimal policies analyzed in the next section. In the decentralized equilibrium, Ally households do not internalize how their portfolio choices affect the outcome of war, creating a wedge between private and social incentives. This externality provides a clear rationale for government intervention to realign private decisions with social objectives. In the next section we show how the Ally's planner has an incentive to distort the decentralized portfolio allocation in order to elicit a security guarantee from the U.S.

## 2.4 Optimal international portfolios policies

In this section, we derive the Ally's optimal international portfolio choice. We assume the planner chooses the country's portfolio, $\{b_1, s_{A,1}, s^*_{U,1}\}$, and implements this allocation via taxes and subsidies on asset holdings that are rebated lump-sum to domestic households. Let $\tau^*_b$ denote a tax (subsidy if negative) on Ally households' holdings of U.S. bonds. Similarly, let $\tau^*_U$ be a tax on Ally holdings of U.S. shares, and let $\tau^*_A$ denote a subsidy on the issuance of Ally shares to U.S. households. The taxes and subsidies that implement the chosen portfolio satisfy

$$1 + \tau^*_b = \frac{\mathbb{E}_0[e_1]}{e_0},$$
$$1 + \tau^*_A = \frac{e_0}{\mathbb{E}_0[e_1]} \frac{\mathbb{E}_0\left[e_1\left(\gamma_1 + 1 + \frac{\alpha^*_1 + 1}{e_1}\right)\right]}{q^*_{A,0}},$$
$$1 + \tau^*_U = \frac{e_0}{\mathbb{E}_0[e_1]} \frac{\mathbb{E}_0\left[e_1\left(2 + \frac{\gamma^*_1 + 1}{e_1}\right)\right]}{q^*_{U,0}},$$

where $e_0$ is determined by the U.S. time-0 budget constraint:

$$\frac{1}{e_0} = 1 + b_1 + \frac{q^*_{A,0}}{e_0} s_{A,1} - q_{U,0} s^*_{U,1}.$$

We begin with a simplified case in which Ally households cannot invest in U.S. shares ($\psi^* \uparrow \infty$), and the planner chooses only $b_1$ and $s_{A,1}$.

**Proposition 5** (Ally optimal policy)**.** *When the Ally faces a security threat ($\rho > 0$), the optimal capital account policy subsidizes the issuance of Ally shares to U.S. households ($\tau^*_A > 0$) and subsidizes domestic holdings of U.S. bonds ($\tau^*_b < 0$). In the absence of a security threat ($\rho = 0$), the decentralized equilibrium is efficient and requires no intervention ($\tau^*_b = \tau^*_A = 0$).*

This proposition is a central result of the paper and shows how the Ally strategically distorts its international portfolio to secure a military guarantee from the United States. The policy has two components. First, the Ally subsidizes the issuance of its shares to U.S. households to increase U.S. exposure to the potential conflict, thereby strengthening U.S. incentives to intervene. While the benefits are clear, there is an associated cost: to induce U.S. households to hold Ally shares, the Ally must offer a higher expected return. This excess return compensates U.S. investors for the costs of investing abroad $\psi$and represents the price the Ally pays to secure the U.S. security guarantee. The optimal subsidy balances the cost of this a wealth transfer cost against the benefit of the security guarantee.

Second, the Ally seeks to influence its net international investment position by encouraging domestic saving through a subsidy on U.S. bond holdings. This policy pushes domestic bond holdings beyond the level that households would otherwise choose, given the concurrent incentives to issue Ally shares. By reducing the U.S. net foreign asset position, the Ally raises the United States' marginal utility of wealth at time 1 and thus reinforces its incentive to protect its foreign investments in a conflict. The cost of this policy arises from distorting the intertemporal allocation of consumption: by subsidizing bond holdings, the planner effectively raises the real return faced by domestic households, shifting their consumption–saving choices away from the efficient allocation. The optimal policy equates this distortionary cost with the attendant security benefits.

The Ally's optimal capital account policy in the presence of a security threat has economic implications that extend beyond procuring a security guarantee. The following corollary summarizes these implications.

**Corollary 6** (Economic implications)**.** *Relative to the decentralized equilibrium, under the Ally's optimal capital account policy:*

*i) The U.S. earns an excess return on its foreign portfolio;*

*ii) The U.S. dollar appreciates, while the U.S. trade balance shifts into a deficit;*

*iii) The Ally's military expenditure decreases, while the U.S.'s military expenditure increases;*

*iv) Both U.S. and Ally households are better off.*

The higher expected return offered by the Ally on its equity, together with the saving incentives embedded in the optimal capital account policy, rationalizes both the appreciation of the dollar and the emergence of a U.S. trade deficit at time 0. As explained in Proposition 3 and 4, greater prospective U.S. engagement in the event of war enables the Ally to reduce its own military spending, while the United States correspondingly increases its expenditure. Despite this reallocation, both countries are better off: the Ally gains from the security guarantee, which strengthens its position in the event of conflict, and the United States benefits from the wealth transfer engineered by the Ally's policy.

# 3 Empirical Evidence

In this section, we present empirical evidence on the critical implications of our model.[9] We focus on Propositions 1 and 2, which yield the sharpest testable predictions: Proposition

[9] The evidence we present is predictive rather than causal. Establishing causal effects is still a work in progress.

1 links U.S. military deployment to bilateral FDI positions, and Proposition 2 links allied defense expenditure to the same positions. Proposition 4 serves as a theoretical benchmark and is not directly testable, while Proposition 5's implications for exorbitant privilege and global imbalances rest on Propositions 1 and 2 holding in the data. For each proposition, we discuss the testable predictions and research design, then report results. Details are in the appendix.

## 3.1 Data

We use a panel dataset of U.S. military deployments and bilateral FDI positions covering two different periods: the Cold War era from 1950 to 1989 when the Berlin Wall fell, and the Globalization Era from 1990 to 2016. We proxy time-one military intervention ($m_1$) using active-duty U.S. military personnel stationed in each country from the Department of Defense Base Structure Reports. This variable measures troop deployment and better directly maps to time-zero military capacity investment rather than actual military intervention, but the model predicts that total military resources invested at time zero ($a_1$) and military resources devoted specifically to the Ally at time 1 ($m_1$) respond to portfolio incentives with the same sign ($\partial a_1/\partial s_{A,1} = \partial m_1/\partial s_{A,1}$).[10] So, troop deployment in a country is a reasonable proxy for preparedness to to intervene in a country at time one.

We use FDI variables from the Bureau of Economic Analysis (BEA), combining historical and current data as detailed in the Appendix. The variables are: (i) FDI in the U.S. from country $i$ (*FDIUS*), and (ii) U.S. direct investment abroad in country $i$ (*USDIA*). Both variables are on a historical-cost basis, scaled as percentages of U.S. GDP.

Due the small number of countries for which *FDIUS* is available since 1950s and the major geopolitical structural break brought about by the end of the Cold War we run the analysis in two sub-samples. The Cold War period includes 10 countries with data since 1960 and two countries with data from 1982. The countries are Belgium (1963), Canada (1951),

[10] In fact, Proposition 3 establishes that initial capacity investment $a_0$ and conditional intervention effort $m_1$ move together: both increase with the U.S. gross asset position and decrease with its net creditor position.

France (1963), Germany (1963), Italy (1963), Netherlands (1951), United Kingdom (1951)—which are NATO members by 1989— and Japan (1960), Sweden (1963), and Switzerland (1951), which are not. Countries with data from 1982 are Israel and Panama. The second sample includes all 35 OECD countries in Table A1 in the Appendix pls 24 other emerging and developing countries (59 countries total).

## 3.2 FDIs and U.S. Military Intervention

This section investigates the implications of Proposition 1. Proposition 1 makes two predictions about U.S. military intervention and financial exposure. The first prediction (an extensive margin) states that the U.S. intervenes militarily if and only if it holds positive equity claims in the Ally (i.e., $s_{A,1} > 0$). Without this exposure, or "skin in the game," the marginal benefit of military effort is zero, so the U.S. does not deploy troops ($m_1 = 0$). In this regard, we simply observe that, in our unbalanced panel data set, more than 80% of country-year observations feature U.S. troop presence during this Cold War period.

The second result characterizes an intensive margin: conditional on positive exposure, U.S. military effort to affect the result of the war that materialized, increases in bilateral gross positions and decreases in the net foreign asset position vis-à-vis the ally. The gross-position channel reflects direct stakes in the ally's productive capacity ($s_{A,1} > 0$): greater equity holdings raise the U.S. incentive to deploy resources protecting those assets. The net-position channel operates through exchange rate hedging: for a given gross position, dollar appreciation following a negative shock to the ally partially hedges U.S. losses, reducing the marginal benefit of additional intervention. The model therefore predicts that Ally holdings in the U.S. (USFDIs) unambiguously increase intervention incentives via the hedging channel, while outward U.S. direct investments (USDIAs) have ambiguous effects due to offsetting "skin in the game" and "hedging" forces. However, in practice, the "skin in the game" effect should be much stronger than the "hedging" effect.

We also note that the model's mechanism—portfolio incentives shaping intervention—

should operate most clearly, but not exclusively, absent explicit security commitments. Formal alliances, such as NATO, create obligations that may substitute for or complement financial incentives. We test whether the FDI-intervention relationship varies across institutional contexts by comparing NATO and non-NATO countries. Throughout, we also control for unobservable country characteristics and time-specific effects, capturing broad trends of financial integration.

The model's timing disciplines our econometric specification. At time 0, countries choose portfolio positions before observing the war realization. At time 1, conditional on war, the U.S. chooses military effort $m_1$ taking portfolios as given. Financial positions thus precede and should predict intervention decisions. Additionally, deployment, our proxy for intervention, should predict the intensity of the intervention. We therefore test the implications of Proposition 1 by estimating the following specification:

$$\text{Troops}_{i,t} = \alpha_i + \beta_1 \text{FDIUS}_{i,t-1} + \beta_2 \text{USDIA}_{i,t-1} + \gamma \mathbf{X}_{i,t-1} + \delta_t + \varepsilon_{i,t} \tag{15}$$

where $\text{Troops}_{i,t}$ measures U.S. active-duty personnel in country $i$ at year $t$, $\alpha_i$ are country fixed effects, and $\delta_t$ are year fixed effects. Proposition 1 implies $\beta_1 > 0$ unambiguously, while $\beta_2$ is theoretically ambiguous. Country fixed effects absorb time-invariant factors including geography and Cold War strategic importance. Year fixed effects control for common shocks such international financial integrations, and the business cycle. Identification exploits within-country variation in FDI positions over time. Standard errors cluster by country.

The error term $\varepsilon_{i,t}$ captures time-varying deployment shocks. In the model, war occurs stochastically with probability $\rho$, and the U.S. responds by choosing $m_1$ conditional on pre-existing portfolios. Empirically, $\varepsilon_{i,t}$ represents year-$t$ surprise in geopolitical threats triggering additional deployment decisions. Our identification strategy requires that, conditional on country and year fixed effects, these shocks are uncorrelated with lagged FDI positions. This assumption would be violated if, for example, investors at $t-1$ systematically anticipate country-specific security deterioration at $t$ and adjust their FDI accordingly. While such anticipation is plausible for slow-moving geopolitical trends, many deployment triggers—coups, border incidents, sudden alliance shifts—occur on shorter timescales than annual FDI reallocation decisions. Country fixed effects absorb persistent heterogeneity in both strategic importance and investment attractiveness, while year fixed effects control for common global shocks, strengthening the conditional exogeneity assumption though not guaranteeing it.

**Table 1** Gross Investment Positions and U.S. Military Deployments 1950–1990

| | (1) Baseline | (2) NATO | (3) Non-NATO | (4) Interactions |
|---|---|---|---|---|
| *Dep. var.:* | U.S. Active-Duty Troops | | | |
| $\text{FDIUS}_{i,t-1}$ | 4,898.3** | 6,824.6* | 4,033.0** | −1,051.2 |
| | (2,408.1) | (3,688.9) | (1,596.7) | (3,923.4) |
| $\text{USDIA}_{i,t-1}$ | 3,050.1 | 1,776.5 | 3,101.8 | 32,462.4*** |
| | (5,299.0) | (5,407.2) | (6,403.4) | (12,444.6) |
| $\text{FDIUS}_{i,t-1} \times$ NATO | | | | 8,453.6* |
| | | | | (4,789.6) |
| $\text{USDIA}_{i,t-1} \times$ NATO | | | | −31,962.4*** |
| | | | | (11,058.3) |
| *Implied effects within NATO:* | | | | |
| FDIUS | | | | 7,402.4 |
| USDIA | | | | 500.0 |
| Observations | 351 | 232 | 119 | 351 |
| Countries | 12 | 7 | 5 | 12 |
| Mean troops | 28,877 | 37,104 | 12,837 | 28,877 |
| Mean FDIUS | 0.203 | 0.254 | 0.117 | 0.203 |
| Mean USDIA | 0.410 | 0.548 | 0.139 | 0.410 |
| Country FE | Yes | Yes | Yes | Yes |
| Year FE | Yes | Yes | Yes | Yes |

*Notes:* OLS estimation with country and year fixed effects. Standard errors clustered by country in parentheses. FDIUS = FDI in the U.S. from country $i$; USDIA = U.S. direct investment in country $i$ (both as % of U.S. GDP, lagged one year). Sample: 12 countries with long historical FDI data from the BEA (i.e., data starting no later than 1982, minimum 8 years). Columns (2)–(3) split by NATO status. $NATO = 1$ from the year of accession on. Column (4): baseline = non-NATO; implied effects = baseline + interaction. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table 1 reports the estimation results. The baseline specifications in columns (1) and provides support for Proposition 1's second prediction. The FDIUS coefficient is positive and statistically significant. This is consistent with the model's unambiguous prediction: an increase in the ally's holdings of U.S. assets raises the gross position (increasing military effort) and worsens the U.S. net position (further increasing effort through higher marginal utility of wealth). In contrast, the USDIA coefficient is positive but statistically indistinguishable from zero, consistent with the theoretically ambiguous prediction for this variable. While higher USDIA raises the U.S. gross asset position (increasing effort), it simultaneously improves the U.S. net creditor position (decreasing effort), and the data suggest these channels approximately offset in aggregate. When we split the sample into NATO vs. non-NATO countries (Columns 2 and 3), the results are ambiguous: the coefficient on FDIUS is larger in the NATO subsample but less precisely estimated than in the non-NATO sample, perhaps reflecting the small number of included countries.

The interaction specification in column (4) reveals substantial heterogeneity in these relationships across alliance types. Outside NATO, the baseline USDIA coefficient is large and highly significant (approximately 32,462 troops per percentage point, p<0.01p < 0.01 p<0.01), while the FDIUS coefficient is small and insignificant. Inside NATO, the pattern reverses: the implied USDIA effect drops to near zero (approximately 500 troops), while the implied FDIUS effect is large and positive (approximately 7,402 troops). This differential response is statistically significant, as evidenced by the highly significant interaction terms.

The different patterns across alliance structures are both economically interpretable and theoretically informative, though they reflect a dimension of heterogeneity not explicitly modeled in our framework. For non-NATO countries, U.S. military deployments respond strongly to direct U.S. asset exposure (USDIA) but not to the ally's financial stake in the United States (FDIUS). This is consistent with Proposition 2's prediction. The significant and large USDIA coefficient outside NATO implies that the gross-position channel—through which greater U.S. asset exposure in the ally strengthens intervention incentives—dominates the offsetting net-position channel in this subsample. Descriptive evidence supports this interpretation: non-NATO countries in our sample exhibit near-balanced FDI positions (average USDIA of 0.14% vs. FDIUS of 0.12% of U.S. GDP), consistent with a weak net creditor position that leaves the gross-position effect largely unopposed. For NATO members, this relationship inverts. The insignificant USDIA effect within NATO is consistent with the net-position channel offsetting the gross-position channel, as predicted by the model in which the effect of USDIA is ambiguous. NATO countries have substantially higher average USDIA than FDIUS (0.55% vs. 0.25% of U.S. GDP), placing the United States in a strong net creditor position where the marginal utility effect dominates and countervails the direct exposure incentive.

The positive and significant FDIUS coefficient within NATO, however, warrants additional interpretation. A possibility here is that NATO's Article 5 commitment fundamentally alters this decision margin. While the model treats intervention as a binary choice followed by an intensity decision, NATO membership may shift the extensive margin (whether to intervene) from financial considerations to treaty obligations, leaving the intensive margin (deployment levels) to be determined by the ally's financial integration into the United States. Under this interpretation, FDIUS becomes a signal of the commitment depth of involvement that justifies higher permanent force postures, while USDIA becomes less relevant because the intervention decision is already contractually determined.

Some limitations warrant acknowledgment. First, the small sample—twelve countries with substantial Cold War troop presence—constrains both statistical power and external validity. The two subsamples raise concerns about whether the estimated coefficients reflect genuine mechanisms or idiosyncratic features of these specific countries. Second, our model provides no formal prediction for how alliance treaties should alter the mapping from financial positions to military deployments. While we can construct ex post rationalizations for why NATO membership reverses the USDIA and FDIUS coefficients, the theory itself is silent on this institutional dimension. Third, the direction of causality remains ambiguous. Lagging

FDI positions are not only consistent with the model's timing but also mitigate concerns about contemporaneous reverse causality (deployments attracting investments). Country and time fixed effects absorb unobservable omitted variables that might jointly determine both FDI and troop deployments. However, forward-looking investors might adjust direct investments in anticipation of future security arrangements, and other unobserved factors (such as evolving strategic importance or bilateral tensions) that could drive both variables. The stark difference between NATO and non-NATO responses provides some reassurance that the patterns reflect genuine economic linkages rather than pure omitted variable bias—it is difficult to construct an omitted variable story that would generate precisely opposite coefficients across alliance types—but we cannot fully rule out alternative explanations.

Despite these limitations, the results provide novel evidence that financial integration can predict military deployments, and that this relationship operates through distinct channels depending on institutional context. The finding that FDIUS predicts deployments within formal alliances, while USDIA predicts deployments outside them, suggests that the theoretical mechanisms identified in Proposition 1—the interplay between gross positions and net creditor status—manifest differently depending on whether intervention decisions are discretionary or treaty-mandated.

## 3.3 FDI and Ally Defense Expenditures

The sample and data construction are identical to those used in Section 3.2. FDI positions (USDIA and FDIUS) are built from BEA historical data and scaled by U.S. GDP. The dependent variable is military expenditure as a share of own GDP from SIPRI. The sample covers 12 countries for which we have both USDIA and FDIUS data: seven NATO members (BEL, CAN, DEU, FRA, GBR, ITA, NLD) and five non-NATO countries (CHE, ISR, JPN, PAN, SWE).

Proposition 2 establishes that the Ally's military expenditure is *increasing* in the U.S. net foreign asset position and, for a given net position, *decreasing* in both gross equity positions

($s_{A,1}$ and $s^*_{U,1}$). To test its implications, we estimate the following specification:

$$\begin{aligned}\text{MilExp}_{i,t} = \alpha_i + \gamma_t + \beta_1\,\text{FDIUS}_{i,t-1} + \beta_2\,\text{USDIA}_{i,t-1} \\ + \beta_3\left(\text{FDIUS}_{i,t-1} \times \text{NATO}_{i,t}\right) + \beta_4\left(\text{USDIA}_{i,t-1} \times \text{NATO}_{i,t}\right) + \varepsilon_{i,t}\end{aligned} \quad (16)$$

where $\text{MilExp}_{i,t}$ is military expenditure as a share of own GDP (SIPRI, %), while all other variables are the same as in Proposition 1. Similarly, identification exploits within-country variation in FDI positions over time, controlling for country fixed effects absorb time-invariant determinants of defense spending (geostrategic importance, institutional characteristics), and year fixed effects that absorb common shocks such as Cold War intensity and the global arms race. Standard errors are clustered by country.

The regression enters USDIA and FDIUS separately, so the coefficient on each variable is interpreted holding the other constant, and since NFA $\approx$ USDIA $-$ FDIUS, the two regressors together span both the gross-position and NFA channels. The predicted sign on FDIUS ($s^*_{U,1}$) is *unambiguously negative*: $\hat{\beta}_1 < 0$. Holding USDIA fixed, an increase in FDIUS lowers the U.S. net foreign asset position, which—by the proposition—raises ally military spending via the NFA channel (*and*) increases the gross U.S. liability position, which by the gross-position channel reduces the ally's incentive to spend on defense. Both effects push in the same direction. This mirrors exactly the structure of Proposition 1, where FDIUS is also the variable with the unambiguous sign for U.S. troop deployment.

**Table 2** GROSS DIRECT INVESTMENTS AND ALLIED MILITARY EXPENDITURE, 1950–1990

| | (1) Baseline | (2) NATO | (3) Non-NATO | (4) Interactions |
|---|---|---|---|---|
| *Panel A: Country Fixed Effects* | | | | |
| $\text{FDIUS}_{i,t-1}$ | $-0.917^{*}$ | $-1.253$ | $-0.281$ | $0.425^{**}$ |
| | (0.534) | (0.810) | (0.469) | (0.204) |
| $\text{USDIA}_{i,t-1}$ | $-2.278$ | $-0.906$ | $-4.478$ | $-2.285^{***}$ |
| | (1.450) | (1.230) | (3.233) | (0.697) |
| $\text{FDIUS}_{i,t-1} \times$ NATO | | | | $-1.712^{***}$ |
| | | | | (0.459) |
| $\text{USDIA}_{i,t-1} \times$ NATO | | | | $-0.006$ |
| | | | | (1.222) |
| *Implied effects within NATO:* | | | | |
| FDIUS | | | | $-1.287$ |
| USDIA | | | | $-2.291$ |
| Observations | 334 | 227 | 107 | 334 |
| Countries | 12 | 7 | 5 | 12 |
| Year FE | No | No | No | No |
| *Panel B: Country and Year Fixed Effects* | | | | |
| $\text{FDIUS}_{i,t-1}$ | 0.093 | $-0.383$ | $2.224^{**}$ | 0.615 |
| | (0.468) | (0.243) | (1.108) | (0.503) |
| $\text{USDIA}_{i,t-1}$ | 0.228 | 0.148 | 0.007 | $3.417^{***}$ |
| | (0.310) | (0.152) | (0.814) | (1.173) |
| $\text{FDIUS}_{i,t-1} \times$ NATO | | | | $-0.513$ |
| | | | | (0.330) |
| $\text{USDIA}_{i,t-1} \times$ NATO | | | | $-3.608^{***}$ |
| | | | | (1.157) |
| *Implied effects within NATO:* | | | | |
| FDIUS | | | | 0.102 |
| USDIA | | | | $-0.192$ |
| Observations | 334 | 227 | 107 | 334 |
| Countries | 12 | 7 | 5 | 12 |
| Year FE | Yes | Yes | Yes | Yes |
| Mean MilExp/GDP | 3.514 | 3.616 | 3.298 | 3.514 |
| Mean FDIUS | 0.221 | 0.267 | 0.121 | 0.221 |
| Mean USDIA | 0.426 | 0.558 | 0.147 | 0.426 |

*Notes:* OLS with country FE in both panels; year FE added in Panel B. SE clustered by country. Dep. var.: military expenditure / GDP (SIPRI, %). FDIUS = FDI in the U.S. from country $i$; USDIA = U.S. direct investment in country $i$ (both % of U.S. GDP, lagged one year; synthetic BEA). NATO (7): BEL, CAN, DEU, FRA, GBR, ITA, NLD. Non-NATO (5): CHE, ISR, JPN, PAN, SWE. NATO = 1 from accession year (time-varying). Col. (4): baseline = non-NATO; implied = baseline + interaction. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

Within NATO, formal treaty commitments provide an unconditional security guarantee that complements the financial channel. Allies inside the alliance, therefore, face a compounded incentive to reduce military effort: the treaty guarantee lowers the marginal return to own defense spending, and bilateral FDI exposure reinforces this by further raising the expected U.S. commitment. We expect the financial exposure effect to be operative everywhere—but potentially stronger within NATO, where the treaty and financial channels overlap and jointly crowd out allied spending. Outside NATO, where the security guarantee is more discretionary, allied governments may be less willing to reduce own military effort in response to FDI exposure alone, since the U.S. commitment is less assured. We explore this heterogeneity by adding interaction terms to the baseline specification.

Table 2 reports results. Panel A only includes country fixed effects; Panel B adds year fixed effects. The comparison between panels is informative. In Panel A, the FDIUS coefficient in the pooled baseline (column 1) is $-0.917^{*}$—correctly signed and consistent with the unambiguous theoretical prediction, although marginally significant. The USDIA coefficient is $-2.278$ (s.e. 1.450), correctly signed but insignificant, consistent with the theoretically ambiguous prediction for this variable. In Panel B, both coefficients change sign and become insignificant (FDIUS: 0.093; USDIA: 0.228), indicating that the Panel A results may be driven by common time trends rather than within-country variation once year fixed effects are absorbed. Splitting by alliance status in columns (2) and (3) does not generate significant estimates in either panel, with the exception of the non-NATO FDIUS coefficient in Panel B, which is positive and significant ($2.224^{**}$)—a sign opposite to the theoretical prediction. The two subsamples, however, are very small.

Column (4) reports a more parsimonious specification. This is the most informative column. In Panel A, the USDIA coefficient (non-NATO) is $-2.285^{***}$ (s.e. 0.697)—significant, correctly signed, and consistent with skin-in-the-game dominating the NFA channel outside the formal alliance. In Panel A, the FDIUS interaction with NATO is $-1.712^{***}$ (s.e. 0.459), so within NATO FDIUS has a large additional negative effect on Ally's spending, and the implied within-NATO FDIUS coefficient is $-1.287$. The implied within-NATO USDIA effect is $-2.291$, also large. These results, therefore, are largely in line or at least can be reconciled with the implications of Proposition 2.

However, in Panel B where the regression includes country and year FE, the picture changes. The non-NATO USDIA coefficient flips sign to $+3.417^{***}$, although the FDIUS coefficient becomes indistinguishable from zero. At the same time, the USDIA×NATO interaction is $-3.608^{***}$, yielding an implied within-NATO USDIA effect of only $-0.192$, with the FDIUS interaction with NATO losing significance ($-0.513$, s.e. 0.330).

These results suggest that both FDI positions and allied military spending moved together over the postwar decades for reasons common to all countries (secular FDI growth, the general decline in allied defense burdens after the early 1960s), and that Panel A is partly capturing this common trend rather than a clean causal response to bilateral FDI variation. Nonetheless, the small-country sample and the same set of caveats stated about the empirical analysis of Proposition 1 warrant caution in drawing definitive conclusions.

# 4 Conclusions

We develop a general equilibrium model linking international portfolio decisions and military security provision between a hegemon and its allies. A country facing an external security threat finds it optimal to subsidize inward FDI from the hegemon and to accumulate the hegemon's bonds, effectively purchasing a security guarantee through financial integration. The hegemon, in turn, finds it optimal to intervene militarily in proportion to its financial exposure. This implicit contract is mutually beneficial: allies reduce their defense burden while the hegemon earns excess returns on its foreign portfolio—the "exorbitant privilege"—as the equilibrium price of protection.

The model rationalizes the three stylized facts documented in the introduction within a single unified framework. First, the hegemon earns an excess return on its international investment position—the exorbitant privilege—because this return differential is the equilibrium price allies pay for the implicit security guarantee embedded in financial integration. Second, the hegemon is the world's largest net exporter of physical capital because the ally's optimal capital account policy subsidizes inward FDI from the hegemon and accumulates hegemon bonds, generating the observed portfolio asymmetry. Third, allies bear a disproportionately low share of the collective defense burden because burden-sharing asymmetry is the equilibrium outcome of these optimal portfolio policies: allies reduce own military effort as the hegemon's financial exposure commits it to intervene.

We test the model's critical predictions using bilateral data on FDI positions, troop deployments, and military spending during the Cold War era. Consistent with the model, we find that larger gross foreign direct investment positions predict greater U.S. troop deployments in that country, with the effect robust to country and year fixed effects and to sample splits by alliance status. The evidence on allied military expenditure is more mixed: higher U.S. direct investment abroad is associated with lower allied defense spending in specifications with country fixed effects, but this relationship is not robust to the inclusion of year fixed effects, suggesting that common postwar trends in FDI growth and declining defense burdens may account for part of the covariation.

Several extensions are still work in progress. On the theoretical front, we are exploring the U.S.'s optimal capital account policy and working to characterize the Nash equilibrium of the strategic interaction between the U.S. and the Ally, as well as the globally optimal equilibrium. On the empirical side, we are broadening the sample to cover the globalization era (1990–2016) and pursuing causal identification through a shift-share exposure design that exploits predetermined legal-origin and linguistic determinants of the geographic distribution of FDI to construct instruments for bilateral investment positions as in the gravity literature on international portfolios.

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Online Appendix

# ‘Portfolio Choice, Military Protection, and Exorbitant Privilege’

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# A1 Variable Definitions, Data Sources, and Summary Statistics

This Appendix defines all variables used in the empirical analysis and provides their sources. It also reports a complete set of summary statistics.

Historical defense expenditure data are from the Global Military Spending Dataset (GMSD) of Barnum et al. (2024) available here. The income balance of the balance of payments is the difference between the current account and the trade balance from the Macrohistory Database of Jordà et al. (2017) available here.

## A1.1 Data

The country sample includes the 35 OECD members listed in Table 1. The sample deliberately excludes Russia (and the USSR before 1991), China, and India. The included countries are heterogeneous in their degree of political alignment with the United States, as measured, for example, by alignment in United Nations voting records. However, none of them can be considered an enemy as assumed in our model.

**Table A1** Country Sample

| **Country Name** | **ISO3** | **Country Name** | **ISO3** |
|---|---|---|---|
| Australia | AUS | Ireland | IRL |
| Austria | AUT | Iceland | ISL |
| Belgium | BEL | Israel | ISR |
| Canada | CAN | Italy | ITA |
| Switzerland | CHE | Japan | JPN |
| Chile | CHL | South Korea | KOR |
| Colombia | COL | Lithuania | LTU |
| Costa Rica | CRI | Latvia | LVA |
| Czech Republic | CZE | Mexico | MEX |
| Germany | DEU | Netherlands | NLD |
| Denmark | DNK | Norway | NOR |
| Spain | ESP | New Zealand | NZL |
| Estonia | EST | Poland | POL |
| Finland | FIN | Portugal | PRT |
| France | FRA | Slovenia | SVN |
| United Kingdom | GBR | Sweden | SWE |
| Greece | GRC | Turkey | TUR |
| Hungary | HUN | | |

Note. The table shows the list of countries included in our empirical analysis.

Post-WWII defense expenditure data are from SIPRI Military Expenditure Database, available here. Foreign direct investment data are from the U.S. BEA. In particular:

- Foreign Direct Investment (FDI) in the U.S.: Balance of Payments and Direct Investment Position Data: Position on a historical-cost basis (Share of GDP), 1980-2024, the longest series available here.
- U.S. Direct Investment Abroad (DIA): Balance of Payments and Direct Investment Position Data U.S. DIA: Position on a historical-cost basis (Share of GDP), 1982-2024, the longest series available here.

Historical BEA data have been digitized and spliced with the current series.

- For *FDIs in the U.S.* we used "Historical Data, Foreign Direct Investment in the United States, 1950-1979" available from SELECTED DATA ON FOREIGN DIRECT INVESTMENT IN THE UNITED STATES, 1950-79. From this publication, we took:
  - Table A (Page 5 in the .pdf) - Foreign Direct Investment in the United States, 1974: Comparison of Estimates Based on the 1959 and 1974 Benchmark Surveys
  - Table 1 (Pages 7,8 in the .pdf) - Foreign Direct Investment Position in the United States at Year-end
  - Table 9 (Pages 22 to 25 in the .pdf) - Foreign Direct Investment Position in the United States at Year-end 1973
- For *U.S. DIAs* we used "Historical Data, Selected Data on U.S. Direct Investment Abroad, 1950-76" available from SELECTED DATA ON U.S. DIRECT INVESTMENT ABROAD, 1950-76. From this publication, we took:
  - Table A (Page 9 in the .pdf) - U.S. Direct Investment Position Abroad, 1957 and 1966: Comparisons of Series Based on 1950, 1957, and 1966 Benchmark Surveys
  - Table 1 (Pages 13 to 39 in the .pdf) - U.S. Direct Investment Position Abroad, Year end (26 years, from 1950 to 1976).
  - Table 9 (Pages 213 to 215 in the .pdf) - -U.S. Direct Investment Position Abroad at Year-end.

# A2 Theoretical Proofs and Extensions

## Proofs

**Lemma 1.** *There exist $\delta^*$ and $\delta$ such that:*
*1) The Ally deploys all its military resources in case of war at time 1.*
*2) The U.S. does not deploy all its military resources in case of war at time 1.*

*Proof.* Ally households' utility in case of war is

$$U_1^*(war) = t.i.p. + \omega^* \log(\theta_1 y_{NT}^*) + \alpha^* \log(\theta_1 y_A) + \beta\left[\alpha^* \log \frac{y_A + \delta^* a_1^*}{\alpha^* + \gamma e_1(war)} + \gamma^* \log \frac{1}{\gamma^* + \alpha e_1(war)}\right],$$

with $a_1^* = a_0^* - h^*(m_1^*)$, $e_1(war) = \frac{\beta\gamma^*}{\beta\gamma - w_1(war)}$, and $w_1(war) = \beta\gamma - \frac{\beta\gamma\left(1-s_{A,1}\right)+s_{U,1}^*\alpha(1+\beta)-b_1}{1-s_{U,1}^*+s_{A,1}\frac{\alpha^*}{\gamma^*}\frac{\theta_1+\beta}{\beta}}$. The first order condition with respect to $m_1^*$ is

$$\frac{\partial U_1^*(war)}{\partial \theta_1}\frac{\partial \theta_1}{\partial m_1^*} - \frac{\beta\alpha^*\delta^*}{y_A + \delta^* a_1^*}\frac{\partial h^*(m_1^*)}{\partial m_1^*} \geq 0,$$

and $a_1^* \geq 0$, with

$$\frac{\partial U_1^*(war)}{\partial \theta_1} = \frac{\omega^* + \alpha^*}{\theta_1} - \beta\left[\frac{\alpha^* e_1(war)^2}{e_1(war) + \frac{\alpha^*}{\gamma}} + \frac{\gamma^* e_1(war)^2}{e_1(war) + \frac{\gamma^*}{\alpha}}\right]\frac{1}{\beta\gamma^*}\frac{\partial w_1(war)}{\partial \theta_1},$$

$$\frac{\partial w_1(war)}{\partial \theta_1} = \frac{\frac{s_{A,1}\alpha^*}{e_1(war)}}{1 - s_{U,1}^* + s_{A,1}\frac{\alpha^*}{\gamma^*}\frac{\theta_1+\beta}{\beta}}$$

Notice that the negative term in the expression for $\frac{\partial U_1^*(war)}{\partial \theta_1}$ is at its maximum when $s_{A,1} = s_{U,1}^* = 1$, $\rho \downarrow 0$ and $n_0 \uparrow \gamma\beta(1+\beta)$ (such that $e_1(war) \uparrow \infty$). Therefore, to ensure that $\frac{\partial U_1^*(war)}{\partial \theta_1} > 0$ we just need to ensure that

$$\omega^* \log(\theta_1 y_{NT}^*) + \omega^* + \alpha^* \log(\theta_1 y_A) + \alpha^* - \beta\frac{\alpha^* + \gamma^*}{\beta + \theta_1} > 0$$

Since the left-hand side is increasing in $\theta_1$, the inequality is satisfied for all $\theta_1 > \tilde{\theta}$ where $\tilde{\theta}$ solves $\tilde{\theta}\left(y_{NT}^*\right)^{\frac{\omega^*}{\omega^*+\alpha^*}}\left(y_A\right)^{\frac{\alpha^*}{\omega^*+\alpha^*}} = e^{\frac{\beta}{\tilde{\theta}+\beta}\frac{\alpha^*+\gamma^*}{\alpha^*+\omega^*}-1}$. Since $\frac{\partial U_1^*(war)}{\partial \theta_1}\frac{\partial \theta_1}{\partial m_1^*} > 0$, then $\exists \bar{\delta}^* > 0$ such that the first order condition holds with a strict inequality when evaluated at $m_1^* = a_0^*$, for all $\delta^* \in \left[0, \bar{\delta}^*\right)$.

U.S. households' utility in case of war is

$$U_1(war) = t.i.p. + \beta\left[\alpha \log \frac{y_U + \delta a_1}{1 + \frac{\gamma^*}{\alpha e_1(war)}} + \gamma \log\left(\frac{1}{1 + \frac{\alpha^*}{\gamma e_1(war)}}\right)\right]$$

with $a_1 = a_0 - h(m_1)$, $e_1(war) = \frac{\beta\gamma^*}{\beta\gamma - w_1(war)}$, and $w_1(war) = \beta\gamma - \frac{\beta\gamma\left(1-s_{A,1}\right)+s^*_{U,1}\alpha(1+\beta)-b_1}{1-s^*_{U,1}+s_{A,1}\frac{\alpha^*}{\gamma^*}\frac{\theta_1+\beta}{\beta}}$. The first order condition with respect to $m_1$ is

$$\left[\frac{\gamma^* e_1(war)}{e_1(war)+\frac{\gamma^*}{\alpha}} + \frac{\alpha^* e_1(war)}{e_1(war)+\frac{\alpha^*}{\gamma}}\right]\frac{1}{\beta\gamma^*}\frac{\partial w_1(war)}{\partial\theta_1}\frac{\partial\theta_1}{\partial m_1} - \frac{\alpha\delta}{y_U+\delta a_1}\frac{\partial h(m_1)}{\partial m_1} \geq 0,$$

and $a_1 \geq 0$, $m_1 \geq 0$. Assume $h(m_1) = m_1 - \varepsilon \ln\left(1+\frac{m_1}{\varepsilon}\right)$, with $\varepsilon > 0$. This implies $\frac{\partial h(0)}{\partial m_1} = \frac{m_1}{\varepsilon+m_1} = 0$, ensuring that $m_1 \geq 0$ is never a binding constraint. Furthermore, $\exists\bar{\delta} > 0$ such that the first order condition is negative when evaluated at $m_1 = a_0$, for all $\delta \geq \bar{\delta}$. Hence, the U.S. does not exhaust all its military resources. □

***Proof of Proposition 1.*** From the proof of Lemma 1, the optimality condition for the choice of $m_1$ is

$$s_{A,1}\frac{\alpha^*}{\beta\gamma^*}\frac{\frac{\gamma^*}{e_1(war)+\frac{\gamma^*}{\alpha}}+\frac{\alpha^*}{e_1(war)+\frac{\alpha^*}{\gamma}}}{1-s^*_{U,1}+s_{A,1}\frac{\alpha^*}{\gamma^*}\frac{\theta_1+\beta}{\beta}}\frac{\partial\theta_1}{\partial m_1} = \frac{\alpha\delta}{y_U+\delta a_1(war)}\frac{\partial h(m_1)}{\partial m_1}.$$

It is immediately evident that the left-hand side is positive if and only if $s_{A,1} > 0$. To prove the second part of the proposition, let

$$\Theta \equiv s_{A,1}\frac{\alpha^*}{\beta\gamma^*}\frac{\frac{\gamma^*}{e_1(war)+\frac{\gamma^*}{\alpha}}+\frac{\alpha^*}{e_1(war)+\frac{\alpha^*}{\gamma}}}{1-s^*_{U,1}+s_{A,1}\frac{\alpha^*}{\gamma^*}\frac{\theta_1+\beta}{\beta}}$$

denote the marginal utility of improving the conflict's outcome, $\theta_1$. We can use the implicit function theorem to show:

$$\frac{\partial m_1}{\partial z} = \frac{\frac{\partial\Theta}{\partial z}\frac{\partial\theta_1}{\partial m_1}+\left(\frac{\partial\Theta}{\partial\theta_1}\frac{\partial\theta_1}{\partial m_1^*}\frac{\partial\theta_1}{\partial m_1}+\Theta\frac{\partial^2\theta_1}{\partial m_1\partial m_1^*}\right)\frac{\partial a_0^*}{\partial z}+\alpha\left(\frac{\delta}{y_U+\delta a_1(war)}\right)^2\frac{m_1}{\varepsilon+m_1}\frac{\partial a_0}{\partial z}}{-\Theta\frac{\partial^2\theta_1}{(\partial m_1)^2}-\frac{\partial\Theta}{\partial\theta_1}\left(\frac{\partial\theta_1}{\partial m_1}\right)^2+\alpha\left(\frac{\delta}{y_U+\delta a_1(war)}\frac{m_1}{\varepsilon+m_1}\right)^2+\frac{\alpha\delta}{y_U+\delta a_1(war)}\frac{\varepsilon}{\varepsilon+m_1}}$$

for $z = b_1, s_{A,1}, s^*_{U,1}$. It's easy to show that $\Theta\frac{\partial^2\theta_1}{(\partial m_1)^2} \leq 0$ and $\frac{\partial\Theta}{\partial\theta_1}\left(\frac{\partial\theta_1}{\partial m_1}\right)^2 \leq 0$. Therefore, the denominator is positive and the sign of $\frac{\partial m_1}{\partial z}$ is determined by the numerator. Notice that since $b_1$, $s_{A,1}$, and $s^*_{U,1}$ are chosen at time 0, they might also affect the choice of $a_0$ and $a_0^*$, either directly or through their impact on $\theta_1$, through $m_1$ and $m_1^*$. Hence, the additional terms at the numerator. In this proof we take $a_0$ and $a_0^*$ as given and focus on the sign of $\frac{\partial\Theta}{\partial z}$ (noting that $\frac{\partial\theta_1}{\partial m_1} > 0$). The next proposition shows that these additional terms have the same sign of $\frac{\partial\Theta}{\partial z}$.

Now we have

$$\frac{\partial\Theta}{\partial b_1} = -s_{A,1}\alpha^*\frac{\gamma^*\left(\frac{e_1(war)}{e_1(war)+\frac{\gamma^*}{\alpha}}\right)^2+\alpha^*\left(\frac{e_1(war)}{e_1(war)+\frac{\alpha^*}{\gamma}}\right)^2}{\left[\beta\gamma^*\left(1-s^*_{U,1}+s_{A,1}\frac{\alpha^*}{\gamma^*}\frac{\theta_1+\beta}{\beta}\right)\right]^2}$$

Therefore, $\frac{\partial m_1}{\partial b_1} < 0$ if $s_{A,1} > 0$. Then

$$\frac{\partial \Theta}{\partial s_{A,1}} = \alpha^* \frac{\left(1 - s^*_{U,1}\right) \left( \frac{\gamma^*}{e_1(war) + \frac{\gamma^*}{\alpha}} + \frac{\alpha^*}{e_1(war) + \frac{\alpha^*}{\gamma}} \right)}{\beta\gamma^* \left(1 - s^*_{U,1} + s_{A,1} \frac{\alpha^*}{\gamma^*} \frac{\theta_1 + \beta}{\beta}\right)^2}$$
$$- \alpha^* s_{A,1} \frac{\left[\gamma^* \left(\frac{e_1(war)}{e_1(war) + \frac{\gamma^*}{\alpha}}\right)^2 + \alpha^* \left(\frac{e_1(war)}{e_1(war) + \frac{\alpha^*}{\gamma}}\right)^2\right] \left[\frac{\gamma}{\gamma^*} + \frac{1}{e_1(war)} \frac{\alpha^*}{\gamma^*} \frac{\theta_1 + \beta}{\beta}\right]}{\beta\gamma^* \left(1 - s^*_{U,1} + s_{A,1} \frac{\alpha^*}{\gamma^*} \frac{\theta_1 + \beta}{\beta}\right)^2}$$

Therefore, $\frac{\partial m_1}{\partial s_{A,1}} > 0$. Finally,

$$\frac{\partial \Theta}{\partial s^*_{U,1}} = s_{A,1} \alpha^* \frac{\frac{\gamma^*}{e_1(war) + \frac{\gamma^*}{\alpha}} + \frac{\alpha^*}{e_1(war) + \frac{\alpha^*}{\gamma}}}{\beta\gamma^* \left(1 - s^*_{U,1} + s_{A,1} \frac{\alpha^*}{\gamma^*} \frac{\theta_1 + \beta}{\beta}\right)^2}$$
$$= s_{A,1} \alpha^* \frac{\left[\gamma^* \left(\frac{e_1(war)}{e_1(war) + \frac{\gamma^*}{\alpha}}\right)^2 + \alpha^* \left(\frac{e_1(war)}{e_1(war) + \frac{\alpha^*}{\gamma}}\right)^2\right] \left(\frac{\alpha}{\gamma^*} \frac{1 + \beta}{\beta} + \frac{1}{e_1(war)}\right)}{\beta\gamma^* \left(1 - s^*_{U,1} + s_{A,1} \frac{\alpha^*}{\gamma^*} \frac{\theta_1 + \beta}{\beta}\right)^2}$$

Therefore, $\frac{\partial m_1}{\partial s^*_{U,1}} > 0$. □

***Proof of Proposition 2***. The Ally household's utility, as a function of $a^*_0$, is

$$U^*_0 = tip + \alpha^* \log \frac{y_A - x^*\left(a^*_0\right)}{\alpha^* + \gamma e_0} + \rho\beta \left[\omega^* \log\left(\theta_1 y^*_{NT}\right) + \alpha^* \log\left(\theta_1 y_A\right)\right]$$
$$+ \beta^2 \mathbb{E}_0 \left[\alpha^* \log \frac{y_A + \delta^* a^*_1}{\alpha^* + \gamma e_1} + \gamma^* \log \frac{1}{\gamma^* + \alpha e_1}\right]$$

The first order condition yields

$$0 = -\frac{\alpha^*}{y_A - x^*\left(a^*_0\right)} \frac{\partial x^*\left(a^*_0\right)}{\partial a^*_0} + \beta^2 \mathbb{E}_0 \left[\frac{\alpha^* \delta^*}{y_A + \delta^* a^*_1}\right] - \frac{\alpha^* \gamma}{\alpha^* + \gamma e_0} \frac{\partial e_0}{\partial a^*_0} + \rho\beta FOC_{m^*_1}$$

where
$$FOC_{m^*_1} \equiv \frac{\partial U^*_1(war)}{\partial \theta_1} \frac{\partial \theta_1}{\partial m^*_1} - \frac{\beta \alpha^* \delta^*}{y_A + \delta^* a^*_1} \frac{\partial h^*\left(m^*_1\right)}{\partial m^*_1} > 0$$

with
$$\frac{\partial U^*_1(war)}{\partial \theta_1} = \frac{\omega^* + \alpha^*}{\theta_1} - s_{A,1} \frac{\alpha^*}{\gamma^*} \frac{\frac{\alpha^*}{1 + \frac{\alpha^*}{\gamma e_1(war)}} + \frac{\gamma^*}{1 + \frac{\gamma^*}{\alpha e_1(war)}}}{1 - s^*_{U,1} + s_{A,1} \frac{\alpha^*}{\gamma^*} \frac{\theta_1 + \beta}{\beta}}$$

Then...(TBC) □

***Proof of Proposition 3.*** The U.S. households' utility as a function of $a_0$ is

$$U_0 = t.i.p + \alpha \log \frac{y_U - x(a_0)}{\alpha + \frac{\gamma^*}{e_0}} + \beta^2 \mathbb{E}_0 \left[ \alpha \log \left( \frac{y_U + \delta a_1}{1 + \frac{\gamma^*}{\alpha e_1}} \right) + \gamma \log \left( \frac{1}{1 + \frac{\alpha^*}{\gamma e_1}} \right) \right]$$

with $a_1 = a_0 - h(m_1)$. The first order condition is

$$0 = -\frac{\alpha}{y_U - x(a_0)} \frac{\partial x(a_0)}{\partial a_0} + \beta^2 \mathbb{E}_0 \left[ \frac{\alpha \delta}{y_U + \delta a_1} \right] + \frac{\alpha \frac{\gamma^*}{e_0}}{\alpha e_0 + \gamma^*} \frac{\partial e_0}{\partial a_0}$$
$$+ \rho \beta^2 \left[ \underbrace{s_{A,1} \frac{\alpha^*}{\beta \gamma^*} \frac{\frac{\gamma^*}{e_1(war) + \frac{\gamma^*}{\alpha}} + \frac{\alpha^*}{e_1(war) + \frac{\alpha^*}{\gamma}}}{1 - s^*_{U,1} + s_{A,1} \frac{\alpha^*}{\gamma^*} \frac{\theta_1 + \beta}{\beta}} \frac{\partial \theta_1}{\partial m_1} - \frac{\alpha \delta}{y_U + \delta a_1(war)} \frac{\partial h(m_1)}{\partial m_1}}_{FOC=0} \right] \frac{\partial m_1}{\partial a_0}$$

where $\frac{\partial e_0}{\partial a_0} = \mathbb{E}_0 \left[ \frac{\partial e_1}{\partial a_0} \right] = \rho \frac{e_1(war)^2}{\beta \gamma^*} \frac{\partial w_1(war)}{\partial \theta_1} \frac{\partial \theta_1}{\partial m_1} \frac{\partial m_1}{\partial a_0}$. Notice that the last part of the first order condition is equal to zero because it coincides with the first order condition for the choice of $m_1$ (see the previous Proposition). Therefore, the first order condition simplifies to:

$$0 = -\frac{\alpha}{y_U - x(a_0)} \frac{\partial x(a_0)}{\partial a_0} + \beta^2 \mathbb{E}_0 \left[ \frac{\alpha \delta}{y_U + \delta a_1} \right]$$
$$+ \frac{\frac{\alpha \gamma^*}{e_0}}{\alpha e_0 + \gamma^*} \rho \frac{e_1(war)^2}{\beta \gamma^*} \frac{\frac{s_{A,1} \alpha^*}{e_1(war)}}{1 - s^*_{U,1} + s_{A,1} \frac{\alpha^*}{\gamma^*} \frac{\theta_1 + \beta}{\beta}} \frac{\partial \theta_1}{\partial m_1} \frac{\partial m_1}{\partial a_0}$$

Since $\frac{\partial x(0)}{\partial a_0} = 0$, then the problem has an interior solution, ie $a_0 > 0$. Now, use the implicit function theorem to show

$$\frac{\partial a_0}{\partial m_1} = \frac{\beta^2 \rho \left( \frac{\delta}{y_U + \delta a_1} \right)^2}{\alpha \left( \frac{1}{y_U - x(a_0)} \frac{\partial x(a_0)}{\partial a_0} \right)^2 + \frac{\alpha}{y_U - x(a_0)} \frac{\partial^2 x(a_0)}{(\partial a_0)^2} + \beta^2 \mathbb{E}_0 \left[ \alpha \left( \frac{\delta}{y_U + \delta a_1} \right)^2 \right]}$$

where we ignored the last term in the first order condition because around the competitive equilibrium we have $\frac{\partial m_1}{\partial a_0} = 0$. Therefore $\frac{\partial a_0}{\partial m_1} > 0$ and $a_0$ has the same comparative static properties of $m_1$ (see Proposition 2). □

***Proof of Proposition 4.*** The exchange rate at time 1 is

$$e_1 = \frac{\left(1 - s^*_{U,1}\right)\left(\gamma^*_1 + \beta \gamma^*\right) + s_{A,1}\left(\alpha^*_1 + \beta \alpha^*\right)}{s^*_{U,1}\left(\alpha_1 + \beta \alpha\right) + \left(1 - s_{A,1}\right)\left(\gamma_1 + \beta \gamma\right) - b_1}$$

If $s_{A,1} = s^*_{U,1} = b_1 = 0$, then $e_1 = \frac{\gamma_1^* + \beta\gamma^*}{\gamma_1 + \beta\gamma} = \frac{\gamma^*}{\gamma}$ is constant. This implies that

$$cov\left(e_1, \gamma_1 + \beta\gamma + \frac{\alpha_1^* + \beta\alpha^*}{e_1}\right) = 0$$
$$cov\left(e_1, \alpha_1 + \beta\alpha + \frac{\gamma_1^* + \beta\gamma^*}{e_1}\right) = 0$$

verifying that $s_{A,1} = s^*_{U,1} = 0$. It also implies

$$q_0 b_1 = \frac{\gamma^*}{e_0} - \gamma = \gamma - \gamma = 0,$$

verifying that $b_1 = 0$. □

***Proof of Proposition 5.*** TBC □

***Proof of Corollary 6.*** TBC □